Almost Event-Rate Independent Monitoring of Metric Dynamic Logic

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AERIAL

ETH zürich

75 NRP

Big Data
National Research Programme

Setting

Online Monitoring Problem

event stream

property

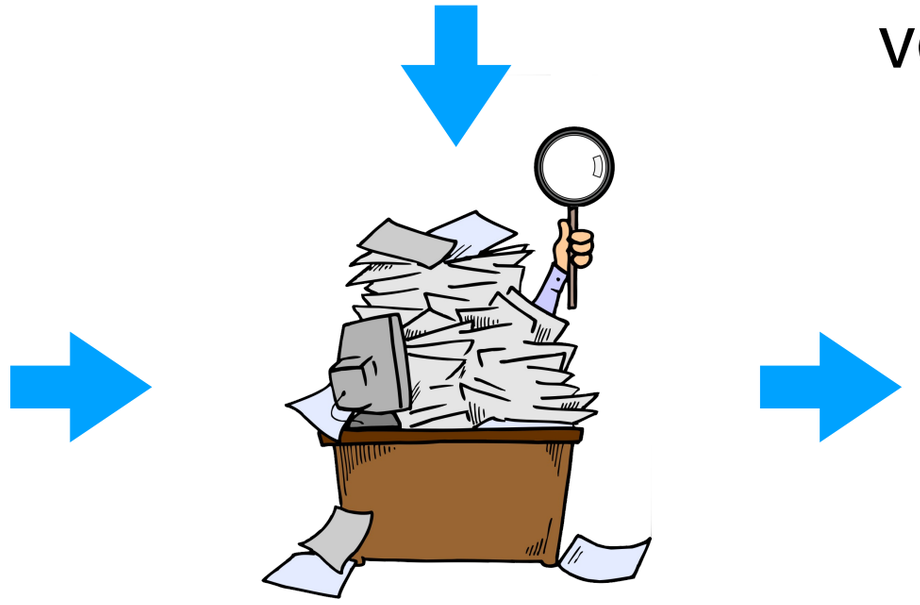
verdict stream

Online Monitoring Problem

within the next 2 time-units both “enter” and “exit” must happen and “enter” must happen before “exit”.

property

event stream



verdict stream

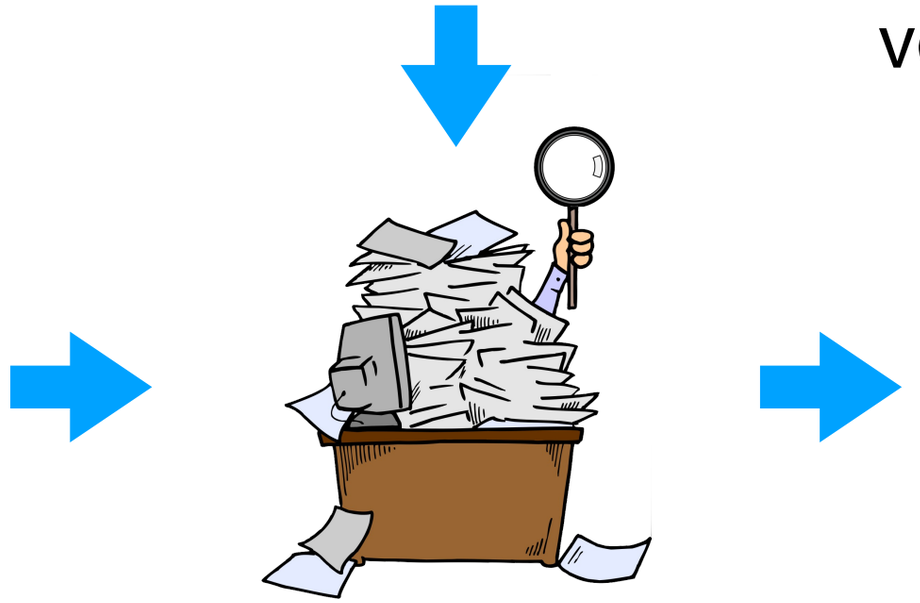
Online Monitoring Problem

within the next 2 time-units both “enter” and “exit” must happen and “enter” must happen before “exit”.

property

event stream

@0



verdict stream

Online Monitoring Problem

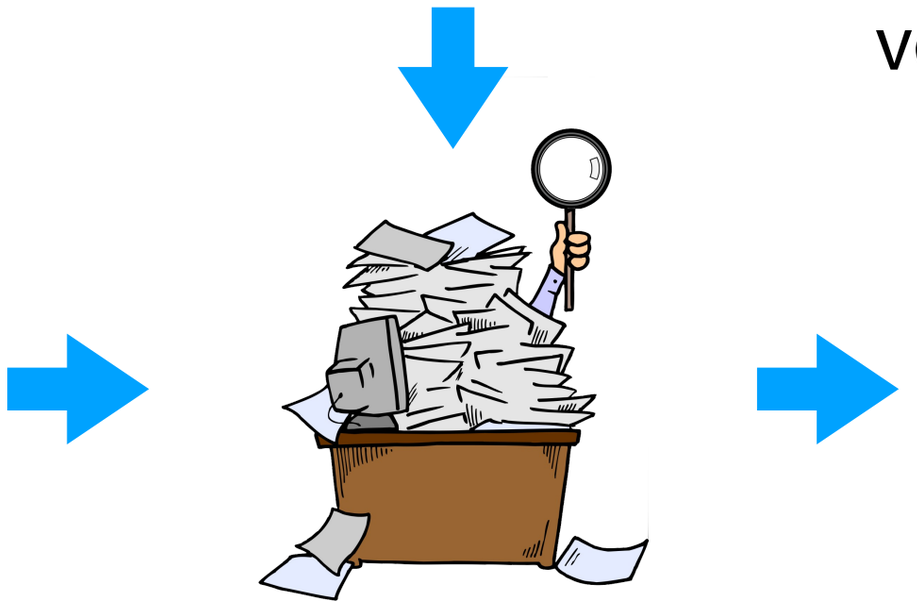
within the next 2 time-units both “enter” and “exit” must happen and “enter” must happen before “exit”.

property

event stream

@0

@1 enter



verdict stream

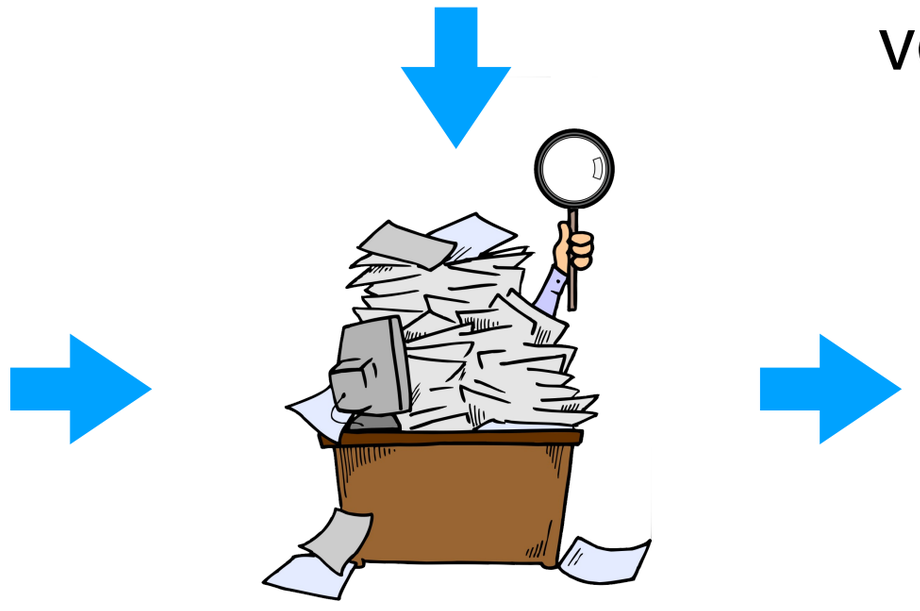
Online Monitoring Problem

within the next 2 time-units both “enter” and “exit” must happen and “enter” must happen before “exit”.

property

event stream

@0
@1 enter
@2 enter exit



verdict stream

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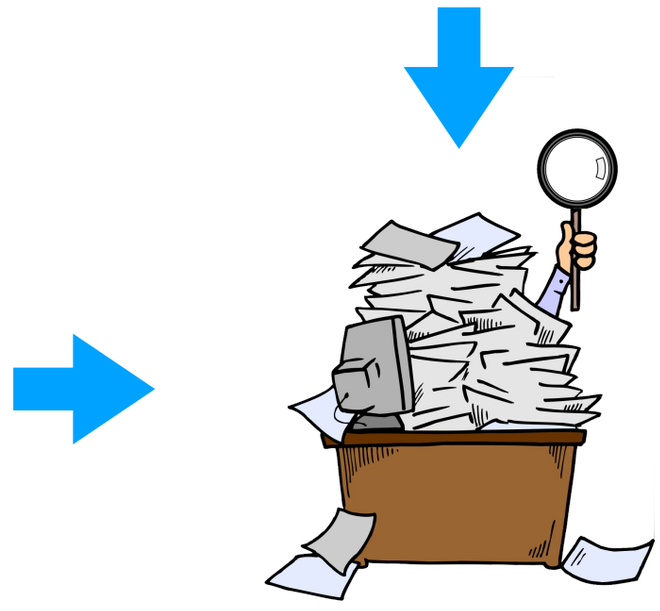


verdict stream

0:0 ✓
1:0 ✓

Online Monitoring Problem

within the next 2 time-units both “enter” and “exit” must happen and “enter” must happen before “exit”.

property

event stream

@0
@1 enter
@2 enter exit
@3 enter

verdict stream

0:0 ✓
1:0 ✓

Online Monitoring Problem

within the next 2 time-units both “enter” and “exit” must happen and “enter” must happen before “exit”.

property

event stream

@0
@1 enter
@2 enter exit
@3 enter
@3 enter

verdict stream

0:0 ✓
1:0 ✓

Online Monitoring Problem

within the next 2 time-units both “enter” and “exit” must happen and “enter” must happen before “exit”.

property

event stream

@0
@1 enter
@2 enter exit
@3 enter
@3 enter
@4 enter

verdict stream

0:0 ✓
1:0 ✓

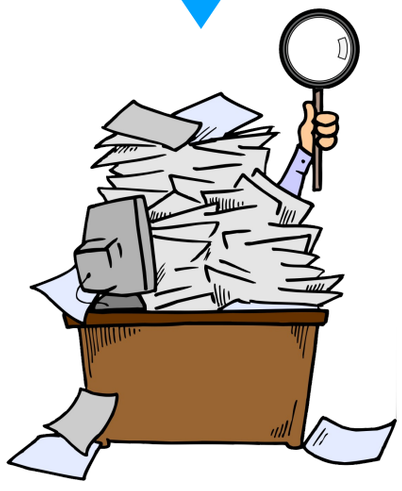
Online Monitoring Problem

within the next 2 time-units both “enter” and “exit” must happen and “enter” must happen before “exit”.

property

event stream

@0
@1 enter
@2 enter exit
@3 enter
@3 enter
@4 enter
@6 exit



verdict stream

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Online Monitoring Problem

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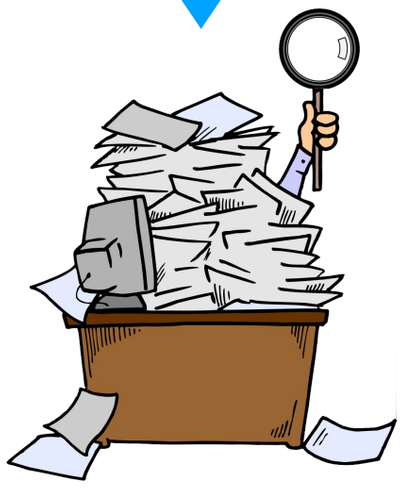
property

event stream

@0
@1 enter
@2 enter exit
@3 enter
@3 enter
@4 enter
@6 exit



verdict stream

0:0 ✓
1:0 ✓
2:0 ✗
3:0 ✗
3:1 ✗
4:0 ✓

Online Monitoring Problem

within the next 2 time-units both “enter” and “exit” must happen and “enter” must happen before “exit”.

property

event stream

@0
@1 enter
@2 enter exit
@3 enter
@3 enter
@4 enter
@6 exit
…

verdict stream

0:0 ✓
1:0 ✓
2:0 ✗
3:0 ✗
3:1 ✗
4:0 ✓

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event stream

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@1 enter
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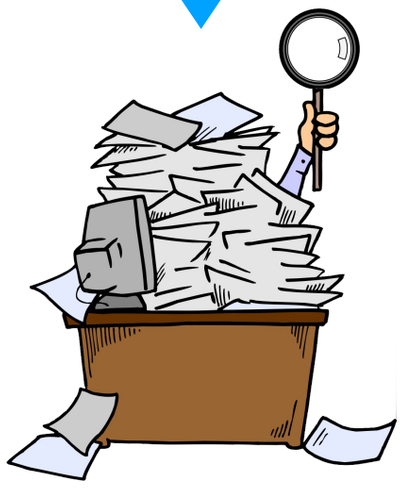
@3 enter
@4 enter
@6 exit
…



verdict stream

0:0 ✓
1:0 ✓
2:0 ✗
3:0 ✗
3:1 ✗
4:0 ✓
…

Online Monitoring Problem

within the next 2 time-units both “enter” and “exit” must happen and “enter” must happen before “exit”.

property

event stream

@0
@1 enter
@2 enter exit
@3 enter
@3 enter
@4 enter
@6 exit
…

verdict stream

0:0 ✓
1:0 ✓
2:0 ✗
3:0 ✗
3:1 ✗
4:0 ✓
…

unlike in CRV: online ≠ instrumentation

Monitoring Output

Monitoring Output

within the next 2 time-units both “enter” and “exit” must happen and “enter” must happen before “exit”.

@0
@1 enter
@2 enter exit
@3 enter
@3 enter
@4 enter
@6 exit
…

0:0 ✓
1:0 ✓
2:0 ✗
3:0 ✗
3:1 ✗
4:0 ✓
…

Monitoring Output

ALWAYS within the next 2 time-units both “enter” and “exit” must happen and “enter” must happen before “exit”.

@0
@1 enter
@2 enter exit
@3 enter
@3 enter
@4 enter
@6 exit
…

within the next 2 time-units both “enter” and “exit” must happen and “enter” must happen before “exit”.

@0
@1 enter
@2 enter exit
@3 enter
@3 enter
@4 enter
@6 exit
…

0:0 ✓
1:0 ✓
2:0 ✗
3:0 ✗
3:1 ✗
4:0 ✓
…

Monitoring Output

ALWAYS within the next 2 time-units both “enter” and “exit” must happen and “enter” must happen before “exit”.

@0
@1 enter
@2 enter exit
@3 enter
@3 enter
@4 enter
@6 exit
…

MARQ

within the next 2 time-units both “enter” and “exit” must happen and “enter” must happen before “exit”.

@0
@1 enter
@2 enter exit
@3 enter
@3 enter
@4 enter
@6 exit
…

MONPOLY

0:0 ✓
1:0 ✓
2:0 ✗
3:0 ✗
3:1 ✗
4:0 ✓
…

Monitoring Output

ALWAYS within the next 2 time-units both “enter” and “exit” must happen and “enter” must happen before “exit”.

@0
@1 enter
@2 enter exit
@3 enter
@3 enter
@4 enter
@6 exit
…

MARQ

✗

within the next 2 time-units both “enter” and “exit” must happen and “enter” must happen before “exit”.

@0
@1 enter
@2 enter exit
@3 enter
@3 enter
@4 enter
@6 exit
…

MONPOLY

0:0 ✓
1:0 ✓
2:0 ✗
3:0 ✗
3:1 ✗
4:0 ✓
…

Monitoring Output

ALWAYS within the next 2 time-units both “enter” and “exit” must happen and “enter” must happen before “exit”.

@0
@1 enter
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...

MARQ

✗

within the next 2 time-units both “enter” and “exit” must happen and “enter” must happen before “exit”.

@0
@1 enter
@2 enter exit
@3 enter
@3 enter
@4 enter
@6 exit
...

MONPOLY

0:0 ✓
1:0 ✓
2:0 ✗
3:0 ✗
3:1 ✗
4:0 ✓
...

Monitoring Output

ALWAYS within the next 2 time-units both “enter” and “exit” must happen and “enter” must happen before “exit”.

@0
@1 enter
@2 enter exit
@3 enter
@3 enter
@4 enter
@6 exit
...

MARQ

✗

within the next 2 time-units both
“e
“e

number unbounded
potentially equal to event-rate

@1 enter
@2 enter exit
@3 enter
@3 enter
@4 enter
@6 exit
...

MONPOLY

0:0 ✓
1:0 ✓
2:0 ✗
3:0 ✗
3:1 ✗
4:0 ✓
...

Monitoring Output

ALWAYS

within the next 2 time-units both “enter” and “exit” must happen and “enter” must happen before “exit”.

@0
@1 enter
@2 enter exit
@3 enter
@3 enter
@4 enter
@6 exit
…

MARQ

✗

within the next 2 time-units both
“e
“e

number unbounded
potentially equal to event-rate

@1 enter
@2 enter exit
@3 enter
@3 enter
@4 enter
@6 exit
…

AERIAL

MONPOLY

0:0 ✓	0:0 ✓
1:0 ✓	1:0 ✓
3:1 = 3:0	2:0 ✗
2:0 ✗	3:0 ✗
3:0 ✗	3:1 ✗
4:0 ✓	4:0 ✓
…	…

Monitoring Output

ALWAYS

within the next 2 time-units both
“enter” and “exit” n
“enter” must happe

@0
@1 enter
@2 enter exit
@3 enter
@3 enter
@4 enter
@6 exit
…

MARQ

✗

number bounded
independent from event-rate

AERIAL

0:0 ✓
1:0 ✓
3:1 = 3:0
2:0 ✗
3:0 ✗
4:0 ✓
…

within the next 2 time-units both
“e
“e

number unbounded
potentially equal to event-rate

@1 enter
@2 enter exit
@3 enter
@3 enter
@4 enter
@6 exit
…

MONPOLY

0:0 ✓
1:0 ✓
2:0 ✗
3:0 ✗
3:1 ✗
4:0 ✓
…

Monitoring Output

ALWAYS

within the next 2 time-units both
“enter” and “exit” n
“enter” must happe

@0
@1 enter
@2 enter exit
@3 enter
@3 enter
@4 enter
@6 exit
…

MARQ

✗

number bounded
independent from event-rate

AERIAL

0:0 ✓
1:0 ✓
3:1 = 3:0
2:0 ✗
3:0 ✗
4:0 ✓
…

index depends
logarithmically
on event-rate

within the next 2 time-units both
“e
“e

number unbounded
potentially equal to event-rate

@1 enter
@2 enter exit
@3 enter
@3 enter
@4 enter
@6 exit
…

MONPOLY

0:0 ✓
1:0 ✓
2:0 ✗
3:0 ✗
3:1 ✗
4:0 ✓
…

Monitoring Output

ALWAYS within the next 2 time-units both “enter” and “exit” n… “enter” must happe…

@0
@1 enter
@2 enter exit
@3 enter
@3 enter
@4 enter
@6 exit
…

MARQ

✗

number bounded
independent from event-rate

AERIAL

0:0 ✓
1:0 ✓
3:1 = 3:0
2:0 ✗
3:0 ✗
4:0 ✓

index depends
logarithmically
on event-rate

within the next 2 time-units both “e… “e…

number unbounded
potentially equal to event-rate

@1 enter
@2 enter exit
@3 enter
@3 enter
@4 enter
@6 exit
…

MONPOLY

0:0 ✓
1:0 ✓
2:0 ✗
3:0 ✗
3:1 ✗
4:0 ✓

almost event-rate independence
[Basin, Bhatt, Traytel, TACAS 2017]

Logic

History

LTL
Pnueli
1977

History

LTL
Pnueli
1977

MTL
Koymans
1990

add notion of time

History

LTL
Pnueli
1977

MTL
Koymans
1990

LDL
De Giacomo, Vardi
2013

RLTL
Leucker, Sánchez
2007

PSL
IEEE 1850
2005

...

add regular expressions

add notion of time

History

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MTL
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LDL
De Giacomo, Vardi
2013

RLTL
Leucker, Sánchez
2007

PSL
IEEE 1850
2005

...

MDL
This paper

add regular expressions

add notion of time

Syntax

$$
\begin{aligned}
\varphi,\psi \;=\; & p \\
& \mid \neg\varphi \mid \varphi \vee \psi \\
& \mid X_I\,\varphi \mid \varphi\, U_I\, \psi \\
& \mid Y_I\,\varphi \mid \varphi\, S_I\, \psi
\end{aligned}
$$

Syntax

$$\varphi, \psi \;=\; p \mid \neg\varphi \mid \varphi \vee \psi \mid \mathsf{X}_I\,\varphi \mid \varphi\,\mathsf{U}_I\,\psi \mid \mathsf{Y}_I\,\varphi \mid \varphi\,\mathsf{S}_I\,\psi$$

atomic

Boolean

future

past

Syntax

$$\varphi, \psi \;=\; p \mid \neg\varphi \mid \varphi \vee \psi \mid \langle r \rangle_I \varphi \mid \varphi \; {}_I\langle r \rangle$$

atomic

Boolean

future

past

Syntax

$$\varphi, \psi \;=\; p \;\mid\; \neg\varphi \;\mid\; \varphi \vee \psi \;\mid\; \langle r \rangle_{\mathrm{I}}\, \varphi \;\mid\; \varphi\, {}_{\mathrm{I}}\langle r \rangle$$

atomic

Boolean

future

past

$$r, s \;=\; \bigstar \;\mid\; \varphi? \;\mid\; r + s \;\mid\; rs \;\mid\; r^*$$

Semantics

$i \models \langle r \rangle_{[a,b]} \varphi$

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$i \models \langle r \rangle_{[a,b]}\ \varphi$ iff there is a j such that

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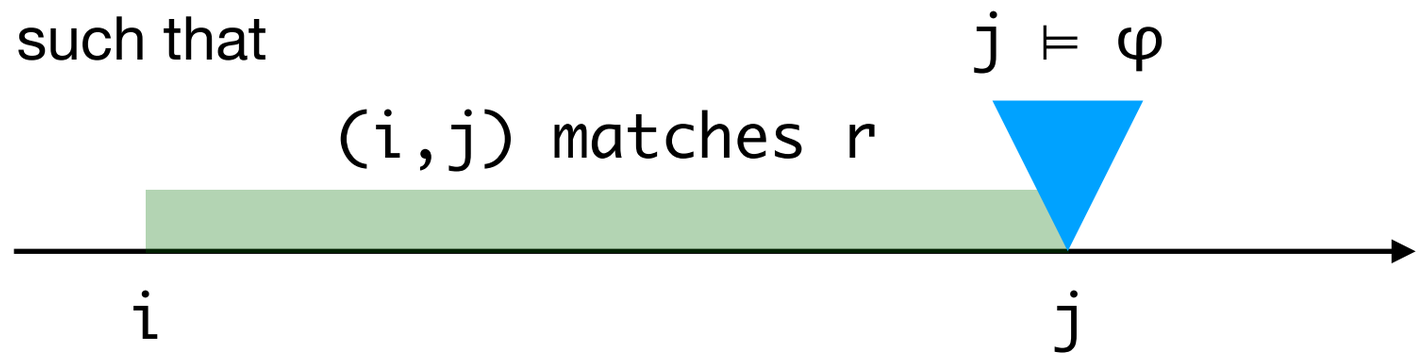

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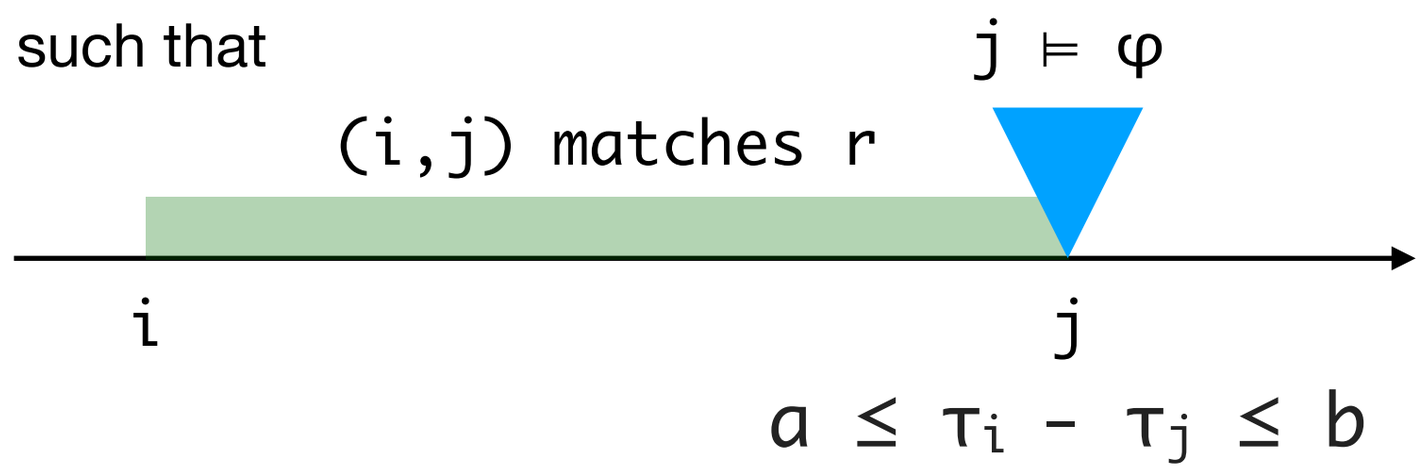

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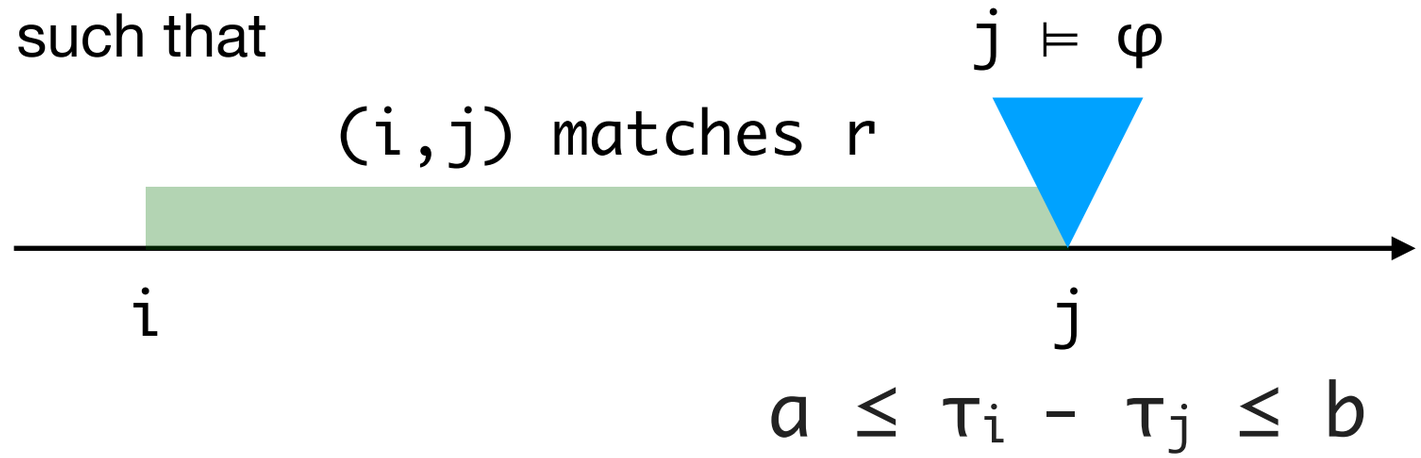
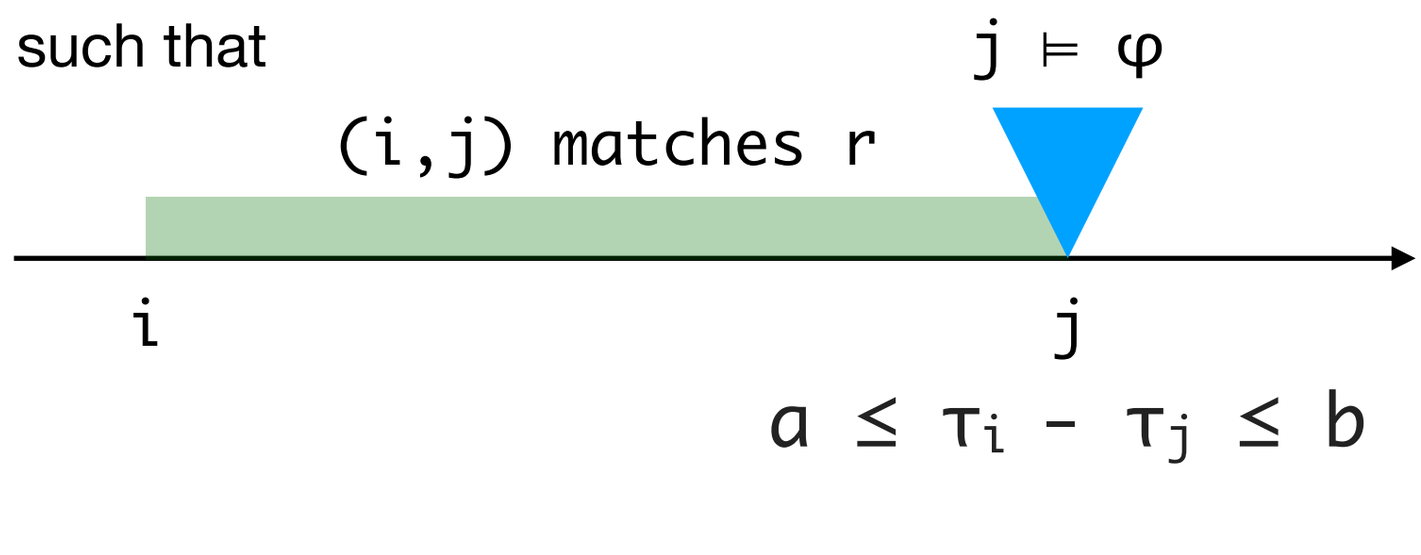


(i,i+1) matches ★

Semantics

$i \models \langle r \rangle_{[a,b]}\ \varphi$ iff there is a j such that

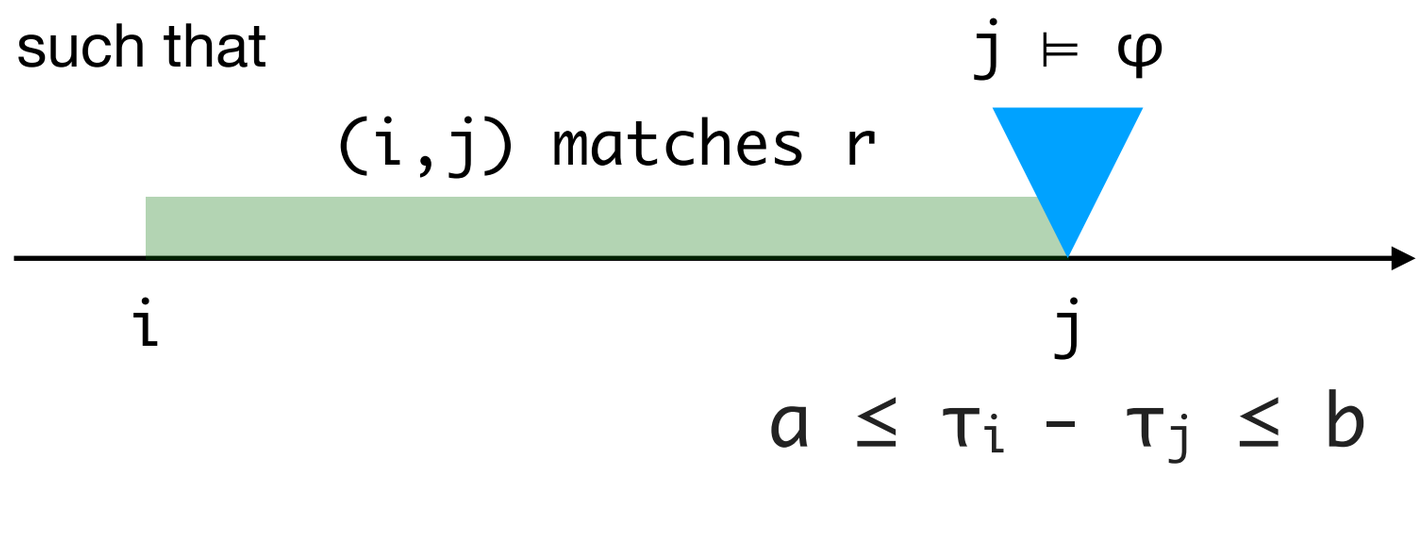


(i,i+1) matches ★

(i,i) matches φ? iff $i \models \varphi$

Semantics

$i \models \langle r \rangle_{[a,b]}\ \varphi$ iff there is a j such that



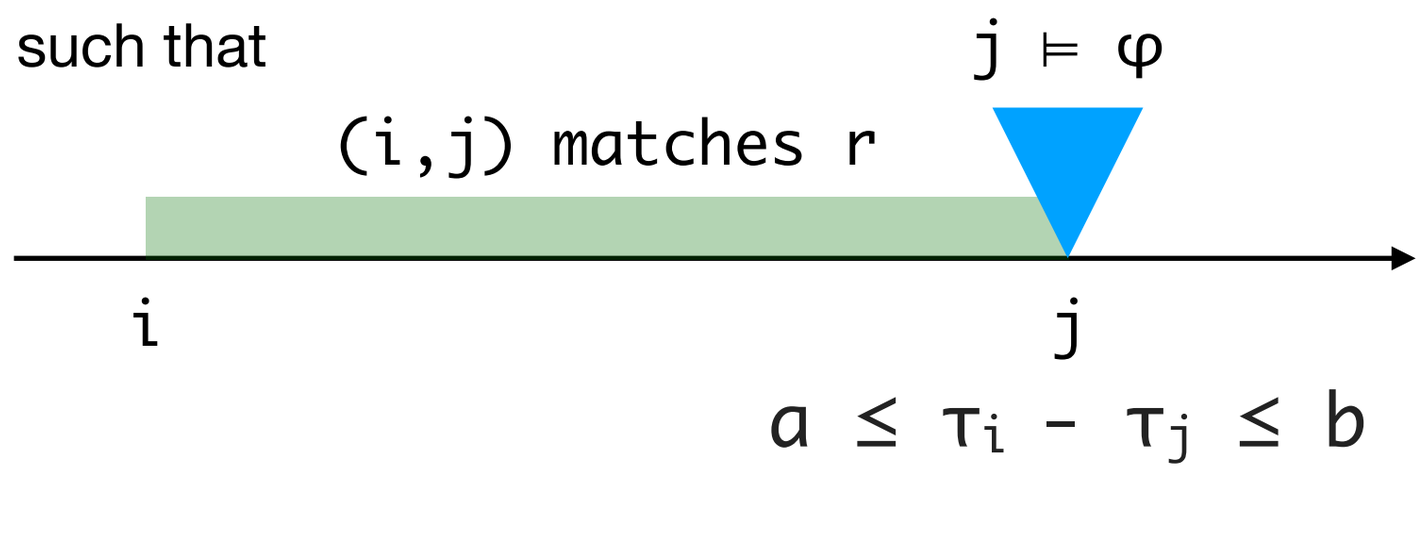
(i,i+1) matches ★

(i,i) matches φ? iff $i \models \varphi$

(i,j) matches r+s iff (i,j) matches r or (i,j) matches s

Semantics

$i \models \langle r \rangle_{[a,b]}\ \varphi$ iff there is a j such that



$(i,i+1)$ matches $\star$

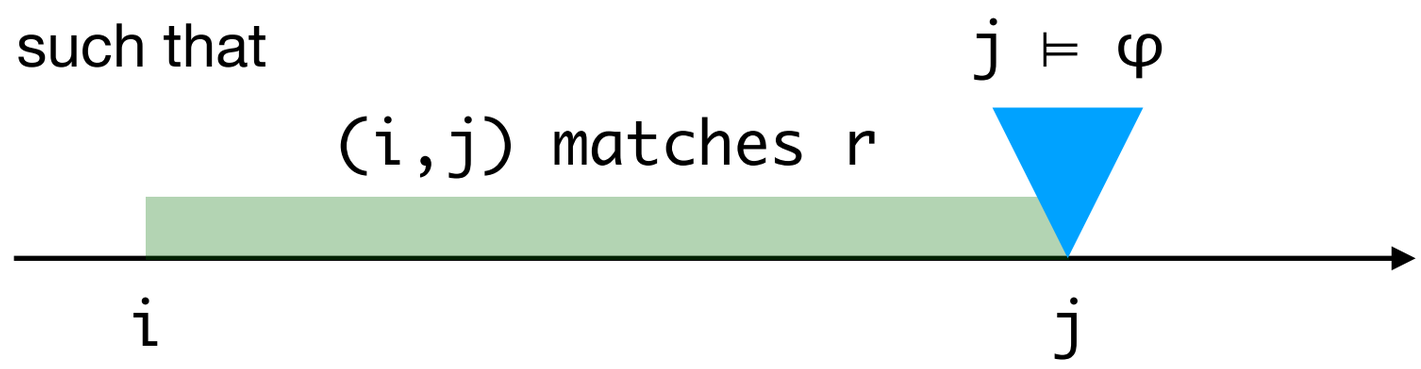
(i,i) matches $\varphi?$ iff $i \models \varphi$

(i,j) matches $r+s$ iff (i,j) matches r or (i,j) matches s

(i,j) matches rs iff there is a k s.t. (i,k) matches r and (k,j) matches s

Semantics

$i \models \langle r \rangle_{[a,b]}\ \varphi$ iff there is a j such that



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(i,j) matches r+s iff (i,j) matches r or (i,j) matches s

(i,j) matches rs iff there is a k s.t. (i,k) matches r and (k,j) matches s

(i,j) matches r* iff …

MTL ⊂ MDL

$\varphi,\psi = p \mid \neg\varphi \mid \varphi \vee \psi \mid \langle r\rangle_I \varphi \mid \varphi\ {}_I\langle r\rangle$
$r,s = \bigstar \mid \varphi? \mid r + s \mid rs \mid r^*$

$X_I\ \varphi$

MTL ⊂ MDL

$\varphi,\psi = p \mid \neg\varphi \mid \varphi \vee \psi \mid \langle r\rangle_I\, \varphi \mid \varphi\, {}_I\langle r\rangle$
$r,s = \bigstar \mid \varphi? \mid r + s \mid rs \mid r^*$

$X_I\, \varphi \quad = \langle\bigstar\rangle_I\, \varphi$

MTL ⊂ MDL

$\varphi,\psi = p \mid \neg\varphi \mid \varphi \vee \psi \mid \langle r\rangle_I\, \varphi \mid \varphi\, {}_I\langle r\rangle$
$r,s = \bigstar \mid \varphi? \mid r + s \mid rs \mid r^*$

$X_I\, \varphi \quad = \langle\bigstar\rangle_I\, \varphi$

$\varphi\ U_I\ \psi$

MTL ⊂ MDL

$$\varphi,\psi = p \mid \neg\varphi \mid \varphi \vee \psi \mid \langle r\rangle_I\, \varphi \mid \varphi\, {}_I\langle r\rangle$$
$$r,s = \bigstar \mid \varphi? \mid r + s \mid rs \mid r^*$$

$$X_I\, \varphi \quad = \langle\bigstar\rangle_I\, \varphi$$

$$\varphi\ U_I\ \psi \ = \langle\varphi^*\rangle_I\, \psi$$

MTL ⊂ MDL

$$\varphi,\psi = p \mid \neg\varphi \mid \varphi \vee \psi \mid \langle r\rangle_I\, \varphi \mid \varphi\, {}_I\langle r\rangle$$
$$r,s = \bigstar \mid \varphi? \mid r + s \mid rs \mid r^*$$

$$X_I\, \varphi \quad = \langle\bigstar\rangle_I\, \varphi$$

$$\varphi\ U_I\ \psi \ = \langle\varphi^*\rangle_I\ \psi \ = \langle(\varphi?\bigstar)^*\rangle_I\ \psi$$

MTL ⊂ MDL

$\varphi,\psi = p \mid \neg\varphi \mid \varphi \vee \psi \mid \langle r\rangle_I\, \varphi \mid \varphi\, {}_I\langle r\rangle$

$r,s = \bigstar \mid \varphi? \mid r + s \mid rs \mid r^*$

$X_I\, \varphi \quad = \langle\bigstar\rangle_I\, \varphi$

$\varphi\ U_I\ \psi \ = \langle\varphi^*\rangle_I\ \psi \ = \langle(\varphi?\bigstar)^*\rangle_I\ \psi$

$Y_I\ \varphi$

MTL ⊂ MDL

$$\varphi,\psi = p \mid \neg\varphi \mid \varphi \vee \psi \mid \langle r\rangle_I\, \varphi \mid \varphi\, {}_I\langle r\rangle$$
$$r,s = \bigstar \mid \varphi? \mid r + s \mid rs \mid r^*$$

$$X_I\, \varphi \quad = \langle\bigstar\rangle_I\, \varphi$$
$$\varphi\ U_I\ \psi \ = \langle\varphi^*\rangle_I\ \psi \ = \langle(\varphi?\bigstar)^*\rangle_I\ \psi$$
$$Y_I\, \varphi \quad = \varphi\ {}_I\langle\bigstar\rangle$$

MTL ⊂ MDL

$\varphi,\psi = p \mid \neg\varphi \mid \varphi \vee \psi \mid \langle r\rangle_I\, \varphi \mid \varphi\, {}_I\langle r\rangle$

$r,s = \bigstar \mid \varphi? \mid r + s \mid rs \mid r^*$

$X_I\, \varphi \quad = \langle\bigstar\rangle_I\, \varphi$

$\varphi\ U_I\ \psi \ = \langle\varphi^*\rangle_I\ \psi \ = \langle(\varphi?\bigstar)^*\rangle_I\ \psi$

$Y_I\, \varphi \quad = \varphi\ {}_I\langle\bigstar\rangle$

$\varphi\ S_I\ \psi$

MTL ⊂ MDL

$$\varphi,\psi = p \mid \neg\varphi \mid \varphi \vee \psi \mid \langle r\rangle_I\, \varphi \mid \varphi\, {}_I\langle r\rangle$$
$$r,s = \bigstar \mid \varphi? \mid r + s \mid rs \mid r^*$$

$$X_I\, \varphi \quad = \langle\bigstar\rangle_I\, \varphi$$
$$\varphi\ U_I\ \psi \ = \langle\varphi^*\rangle_I\ \psi \ = \langle(\varphi?\bigstar)^*\rangle_I\ \psi$$
$$Y_I\, \varphi \quad = \varphi\ {}_I\langle\bigstar\rangle$$
$$\varphi\ S_I\ \psi \ = \psi\ {}_I\langle\varphi^*\rangle$$

MTL ⊂ MDL

$$\varphi,\psi = p \mid \neg\varphi \mid \varphi \vee \psi \mid \langle r\rangle_I\, \varphi \mid \varphi\, {}_I\langle r\rangle$$
$$r,s = \bigstar \mid \varphi? \mid r + s \mid rs \mid r^*$$

$$X_I\, \varphi \quad = \langle\bigstar\rangle_I\, \varphi$$
$$\varphi\ U_I\ \psi \ = \langle\varphi^*\rangle_I\ \psi \ = \langle(\varphi?\bigstar)^*\rangle_I\ \psi$$
$$Y_I\, \varphi \quad = \varphi\ {}_I\langle\bigstar\rangle$$
$$\varphi\ S_I\ \psi \ = \psi\ {}_I\langle\varphi^*\rangle \ = \psi\ {}_I\langle(\bigstar\varphi?)^*\rangle$$

MTL ⊂ MDL

$$\varphi,\psi = p \mid \neg\varphi \mid \varphi \vee \psi \mid \langle r\rangle_I\, \varphi \mid \varphi\, {}_I\langle r\rangle$$
$$r,s = \bigstar \mid \varphi? \mid r + s \mid rs \mid r^*$$

$$X_I\, \varphi \quad = \langle\bigstar\rangle_I\, \varphi$$
$$\varphi\ U_I\ \psi \ = \langle\varphi^*\rangle_I\ \psi \ = \langle(\varphi?\bigstar)^*\rangle_I\ \psi$$
$$Y_I\, \varphi \quad = \varphi\ {}_I\langle\bigstar\rangle$$
$$\varphi\ S_I\ \psi \ = \psi\ {}_I\langle\varphi^*\rangle \ = \psi\ {}_I\langle(\bigstar\varphi?)^*\rangle$$

within the next 2 time-units both “enter” and “exit” must happen and “enter” must happen before “exit”.

MTL ⊂ MDL

$$\varphi,\psi = p \mid \neg\varphi \mid \varphi \vee \psi \mid \langle r\rangle_I\, \varphi \mid \varphi\, {}_I\langle r\rangle$$
$$r,s = \bigstar \mid \varphi? \mid r + s \mid rs \mid r^*$$

$$X_I\, \varphi \quad = \langle\bigstar\rangle_I\, \varphi$$
$$\varphi\ U_I\ \psi \ = \langle\varphi^*\rangle_I\ \psi \ = \langle(\varphi?\bigstar)^*\rangle_I\ \psi$$
$$Y_I\, \varphi \quad = \varphi\ {}_I\langle\bigstar\rangle$$
$$\varphi\ S_I\ \psi \ = \psi\ {}_I\langle\varphi^*\rangle \ = \psi\ {}_I\langle(\bigstar\varphi?)^*\rangle$$

within the next 2 time-units both “enter” and “exit” must happen and “enter” must happen before “exit”.

$$\langle\bigstar^*\, \mathtt{enter}\, \bigstar^*\rangle\ [0,2]\ \mathtt{exit}$$

Algorithm

Dynamic Programming for Past-time LTL

Dynamic Programming for Past-time LTL

[Havelund & Roşu, TACAS 2002]

Dynamic Programming for Past-time LTL

...

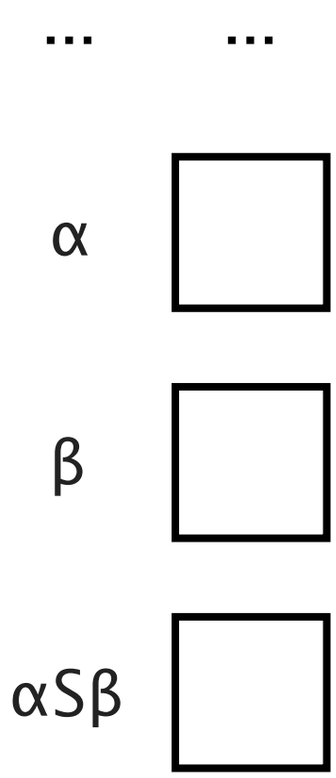
α

β

$\alpha S \beta$

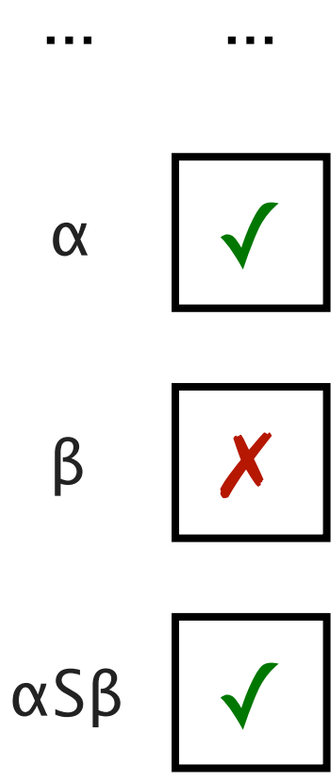
[Havelund & Roşu, TACAS 2002]

Dynamic Programming for Past-time LTL



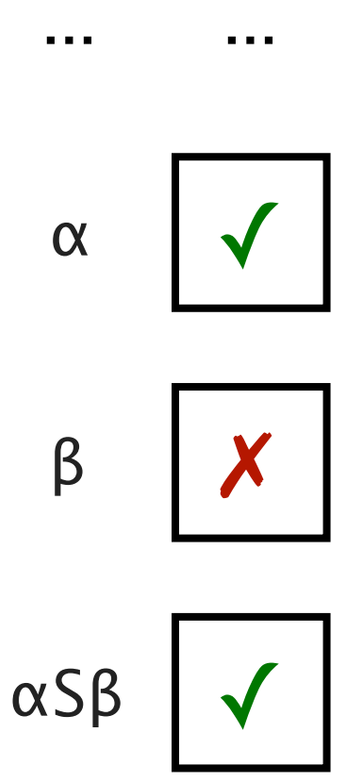
[Havelund & Roşu, TACAS 2002]

Dynamic Programming for Past-time LTL



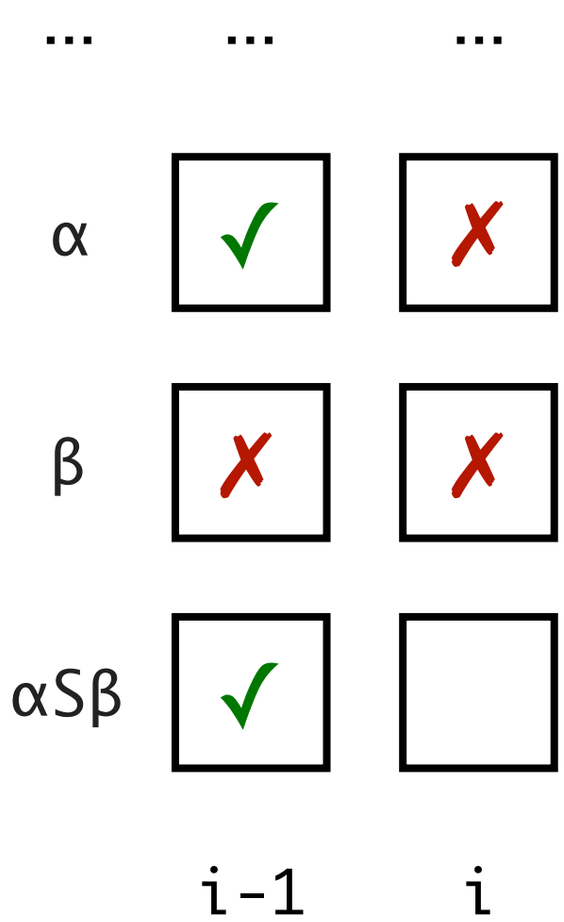
[Havelund & Roşu, TACAS 2002]

Dynamic Programming for Past-time LTL



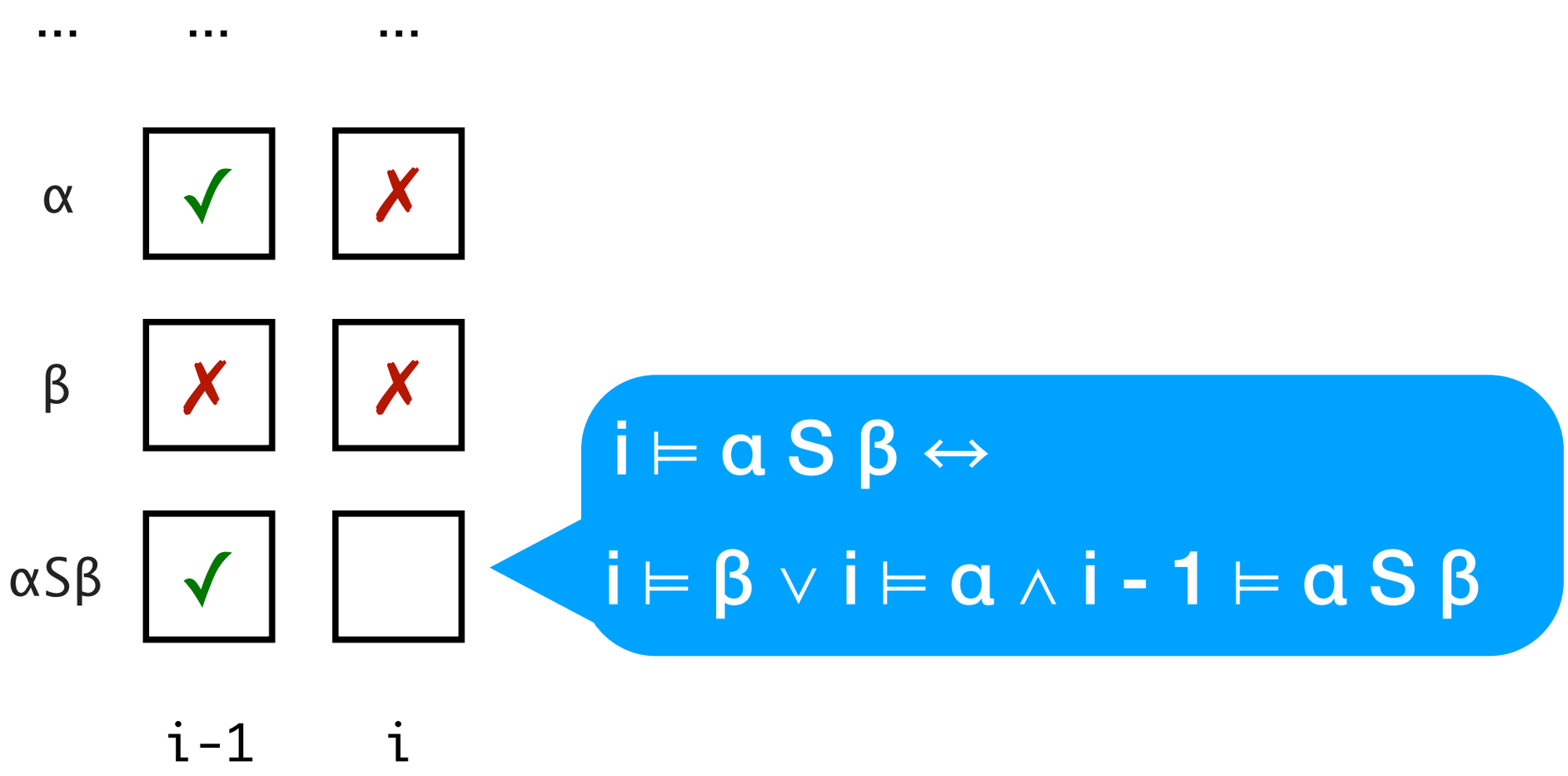
[Havelund & Roşu, TACAS 2002]

Dynamic Programming for Past-time LTL



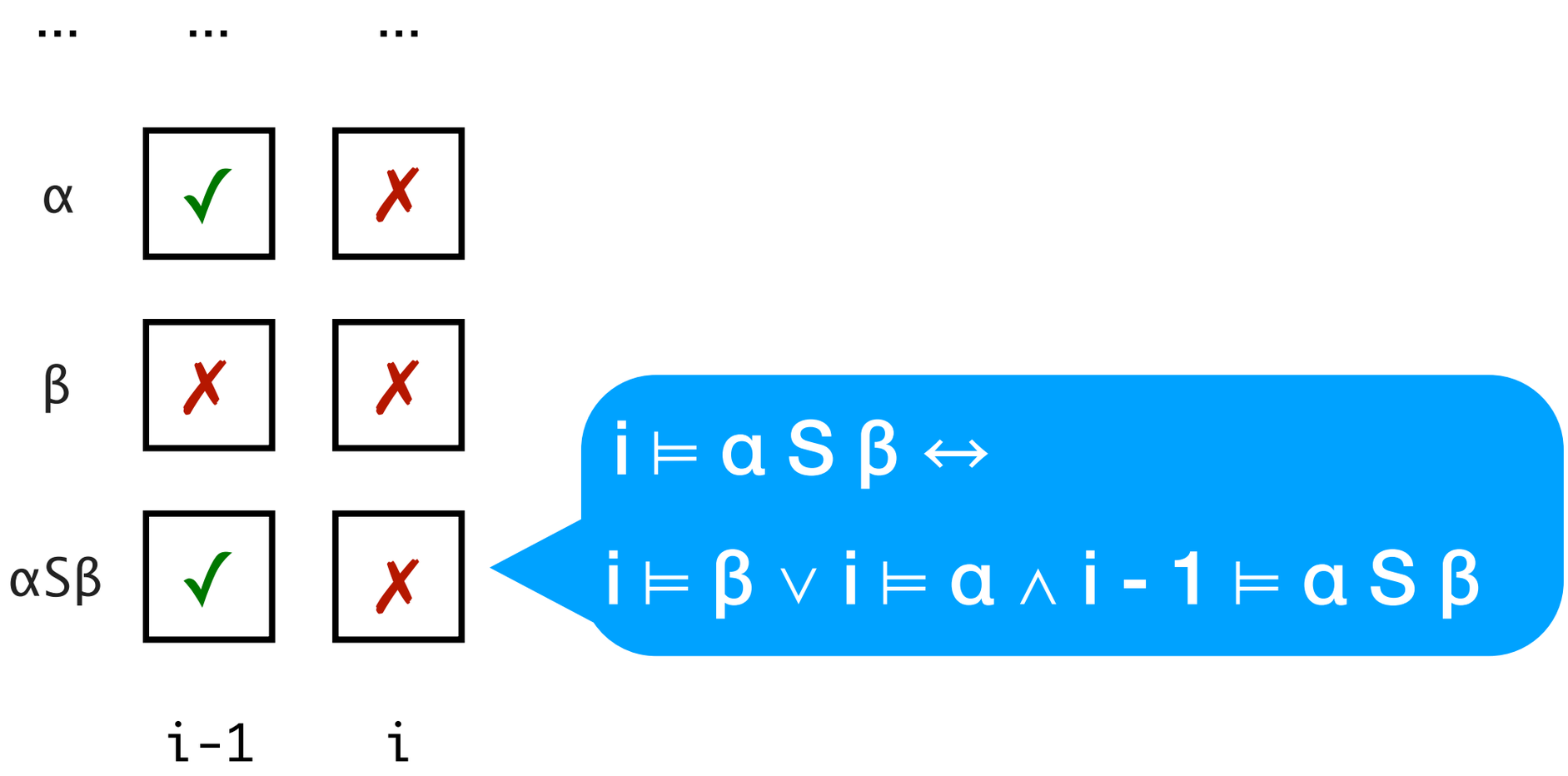
[Havelund & Roşu, TACAS 2002]

Dynamic Programming for Past-time LTL



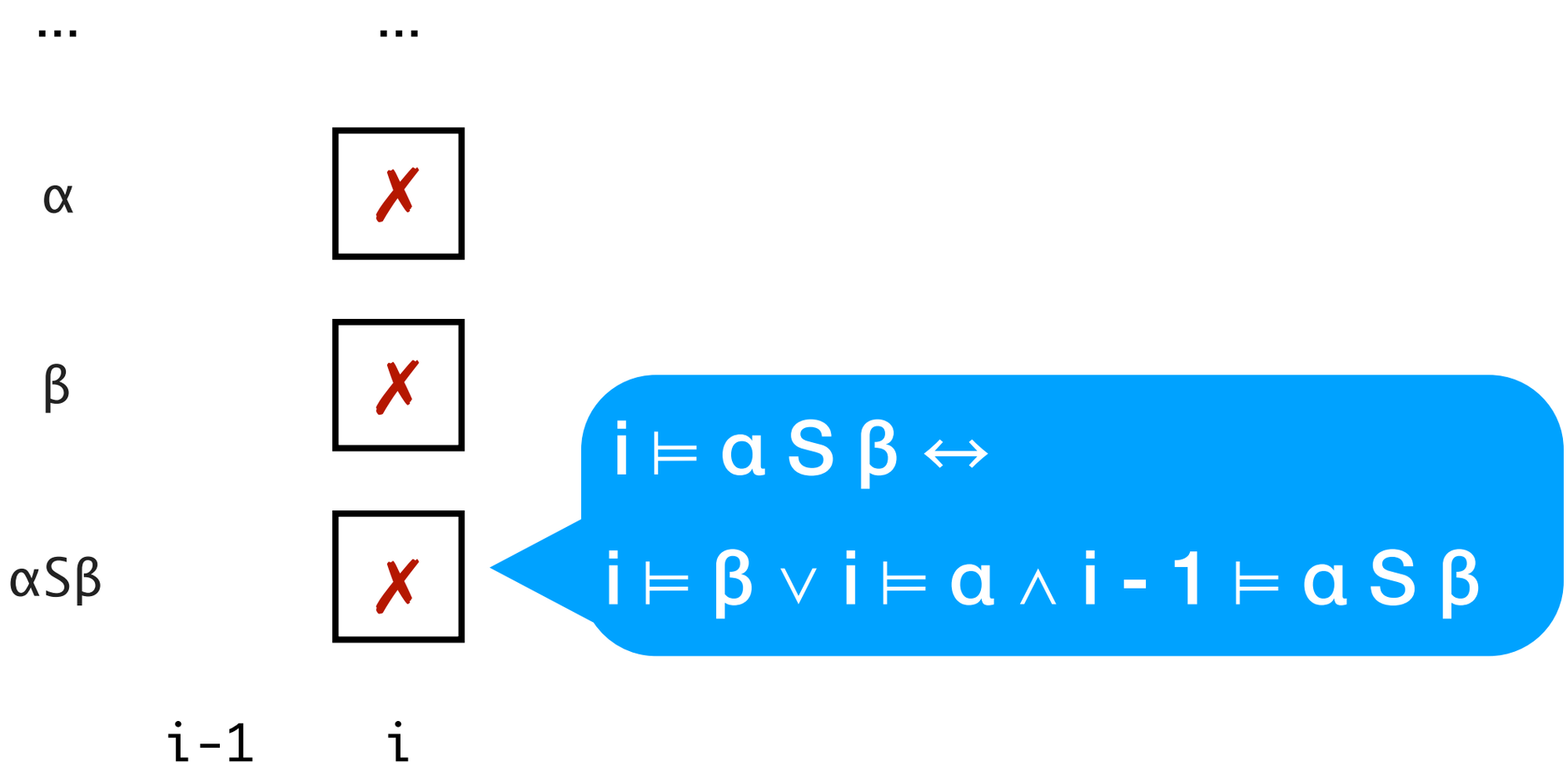
[Havelund & Roşu, TACAS 2002]

Dynamic Programming for Past-time LTL



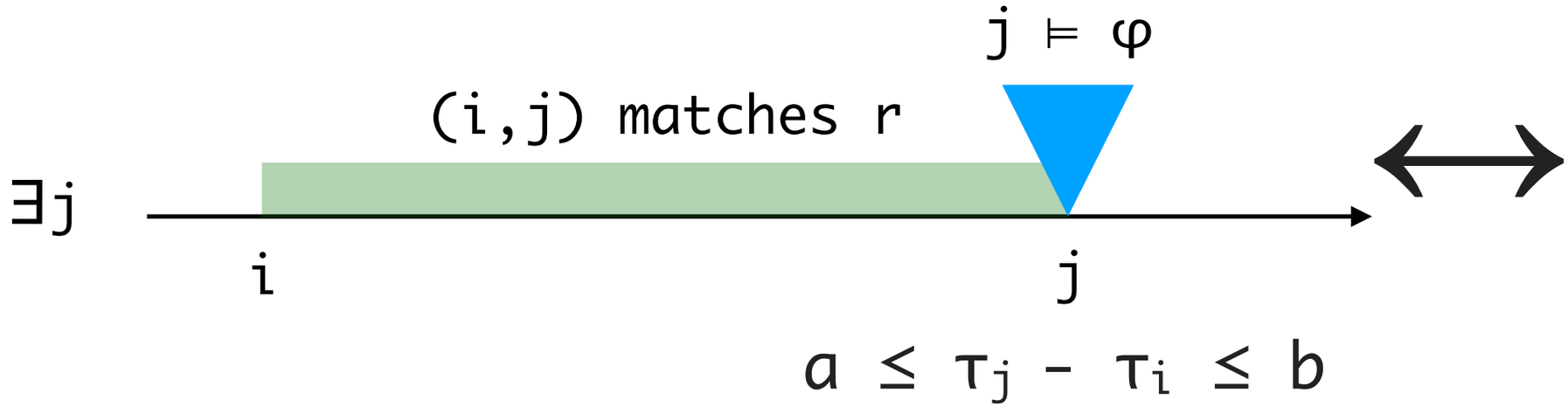
[Havelund & Roşu, TACAS 2002]

Dynamic Programming for Past-time LTL

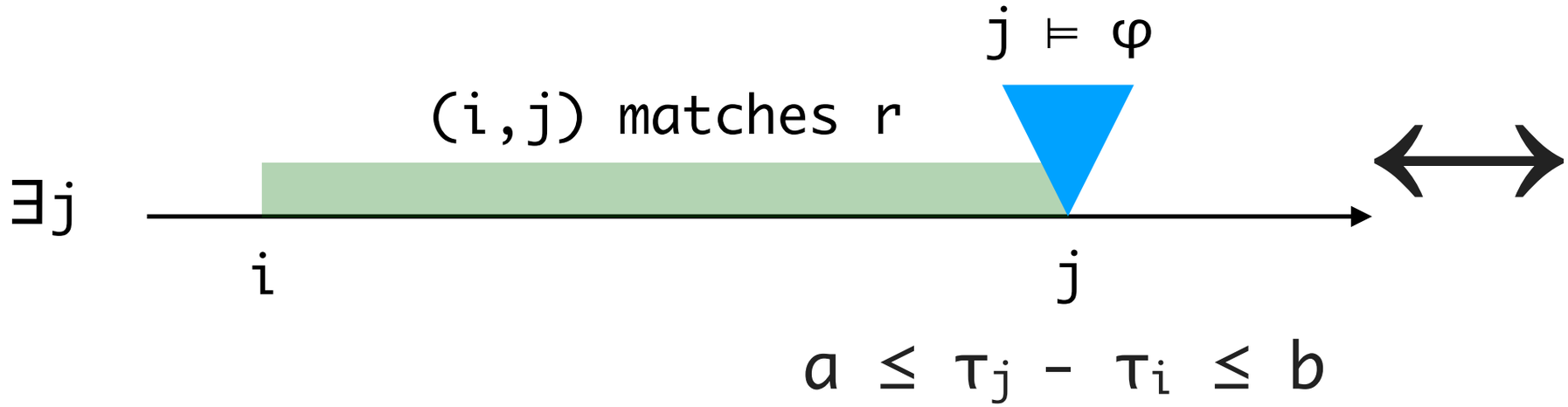


[Havelund & Roşu, TACAS 2002]

Incremental MDL Semantics

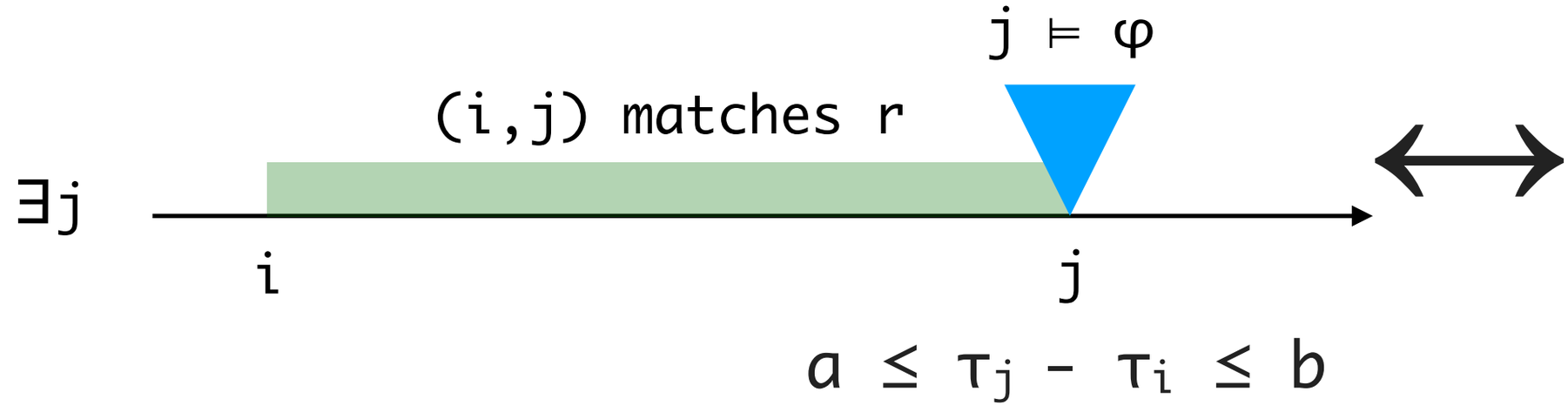


Incremental MDL Semantics

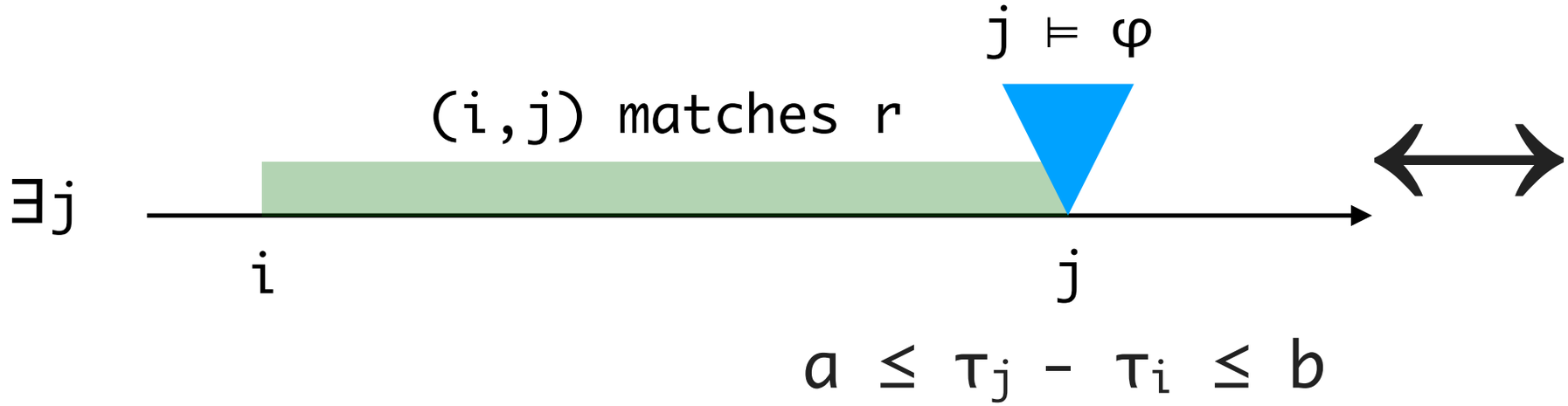


V

Incremental MDL Semantics

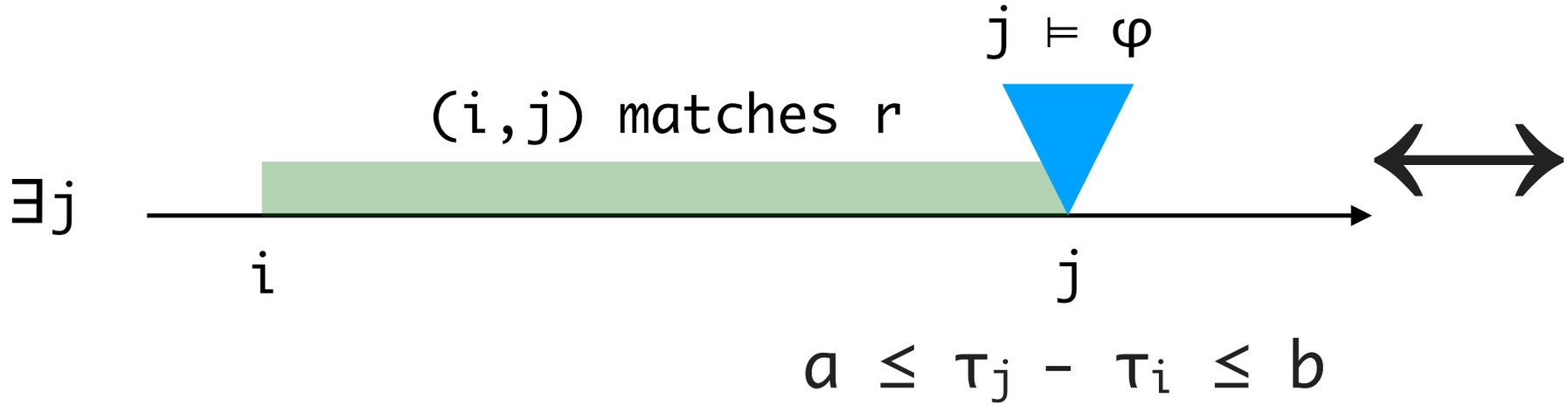


Incremental MDL Semantics



$\Delta := \tau_{i+1} - \tau_i$

Incremental MDL Semantics



$a = 0 \quad \wedge \quad \wedge \quad \bigvee$

$\Delta \leq b \quad \wedge \exists j$

$\Delta := \tau_{i+1} - \tau_i$

$a \leq \tau_j - \tau_i \leq b$

$a - \Delta \leq \tau_j - \tau_{i+1} \leq b - \Delta$

Incremental MDL Semantics

$i \models \langle r \rangle_{[a,b]} \varphi$ $\longleftrightarrow$

$a = 0 \quad \bigwedge \quad i \models \varphi \quad \bigwedge \quad (i,i)$ matches $r \quad \bigvee$

$\Delta \leq b \quad \bigwedge \quad i{+}1 \models \langle ??? \rangle_{[a-\Delta,b-\Delta]} \varphi$

$\Delta := \tau_{i+1} - \tau_i$

Incremental MDL Semantics

$$i \models \langle r \rangle_{[a,b]} \varphi \quad \longleftrightarrow$$

$$a = 0 \quad \wedge \quad i \models \varphi \quad \wedge \quad (i,i) \text{ matches } r \quad \vee$$

$$\Delta \leq b \quad \wedge \quad i{+}1 \models \langle \delta_i(r) \rangle_{[a-\Delta,b-\Delta]} \varphi$$

$$\Delta := \tau_{i+1} - \tau_i$$

Incremental MDL Semantics

$$i \models \langle r \rangle_{[a,b]} \varphi \quad \longleftrightarrow$$

$$a = 0 \quad \wedge \quad i \models \varphi \quad \wedge \quad \varepsilon_i(r) \quad \vee$$

$$\Delta \leq b \quad \wedge \quad i{+}1 \models \langle \delta_i(r) \rangle_{[a-\Delta,b-\Delta]} \varphi$$

$$\Delta := \tau_{i+1} - \tau_i$$

Brzozowski Derivative

$$\varepsilon_i(\star) = \bot \qquad \delta_i(\star) = \top?$$

$$\varepsilon_i(\varphi?) = i \models \varphi \qquad \delta_i(\varphi?) = \bot?$$

$$\varepsilon_i(r + s) = \varepsilon_i(r) \vee \varepsilon_i(s) \qquad \delta_i(r + s) = \delta_i(r) + \delta_i(s)$$

$$\varepsilon_i(r \cdot s) = \varepsilon_i(r) \wedge \varepsilon_i(s) \qquad \delta_i(r \cdot s) = \delta_i(r) \cdot s + \varepsilon_i(r)? \cdot \delta_i(s)$$

$$\varepsilon_i(r^*) = \top \qquad \delta_i(r^*) = \delta_i(r) \cdot r^*$$

Brzozowski Derivative

$$\varepsilon_i(\star) = \bot \qquad \delta_i(\star) = \top?$$

$$\varepsilon_i(\varphi?) = i \models \varphi \qquad \delta_i(\varphi?) = \bot?$$

$$\varepsilon_i(r + s) = \varepsilon_i(r) \vee \varepsilon_i(s) \qquad \delta_i(r + s) = \delta_i(r) + \delta_i(s)$$

$$\varepsilon_i(r \cdot s) = \varepsilon_i(r) \wedge \varepsilon_i(s) \qquad \delta_i(r \cdot s) = \delta_i(r) \cdot s + \varepsilon_i(r)? \cdot \delta_i(s)$$

$$\varepsilon_i(r^*) = \top \qquad \delta_i(r^*) = \delta_i(r) \cdot r^*$$

not the whole story; see paper

MDL Monitor by Example

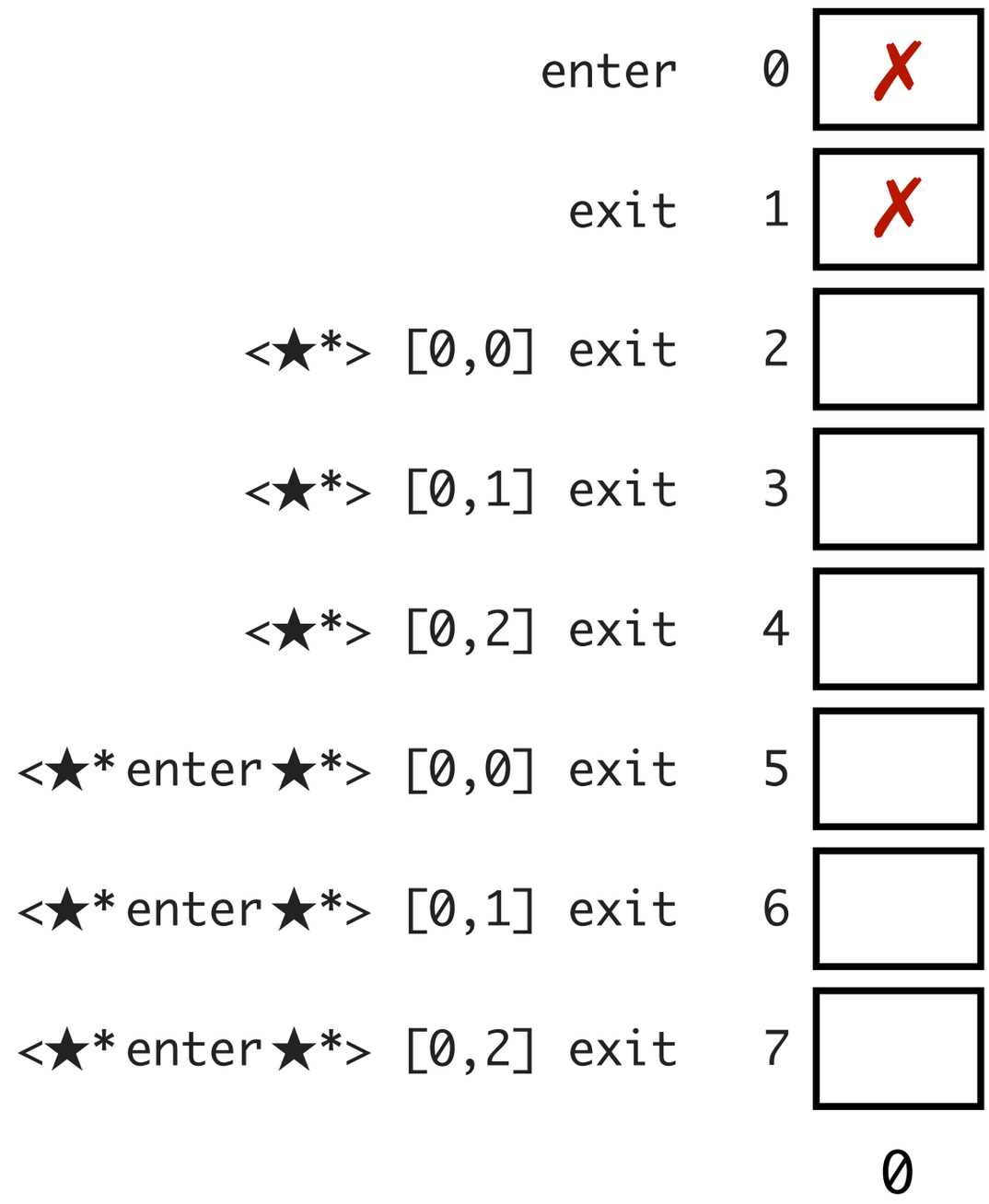
MDL Monitor by Example

```
                  enter
                   exit
      <★*> [0,0] exit
      <★*> [0,1] exit
      <★*> [0,2] exit
<★*enter★*> [0,0] exit
<★*enter★*> [0,1] exit
<★*enter★*> [0,2] exit
```

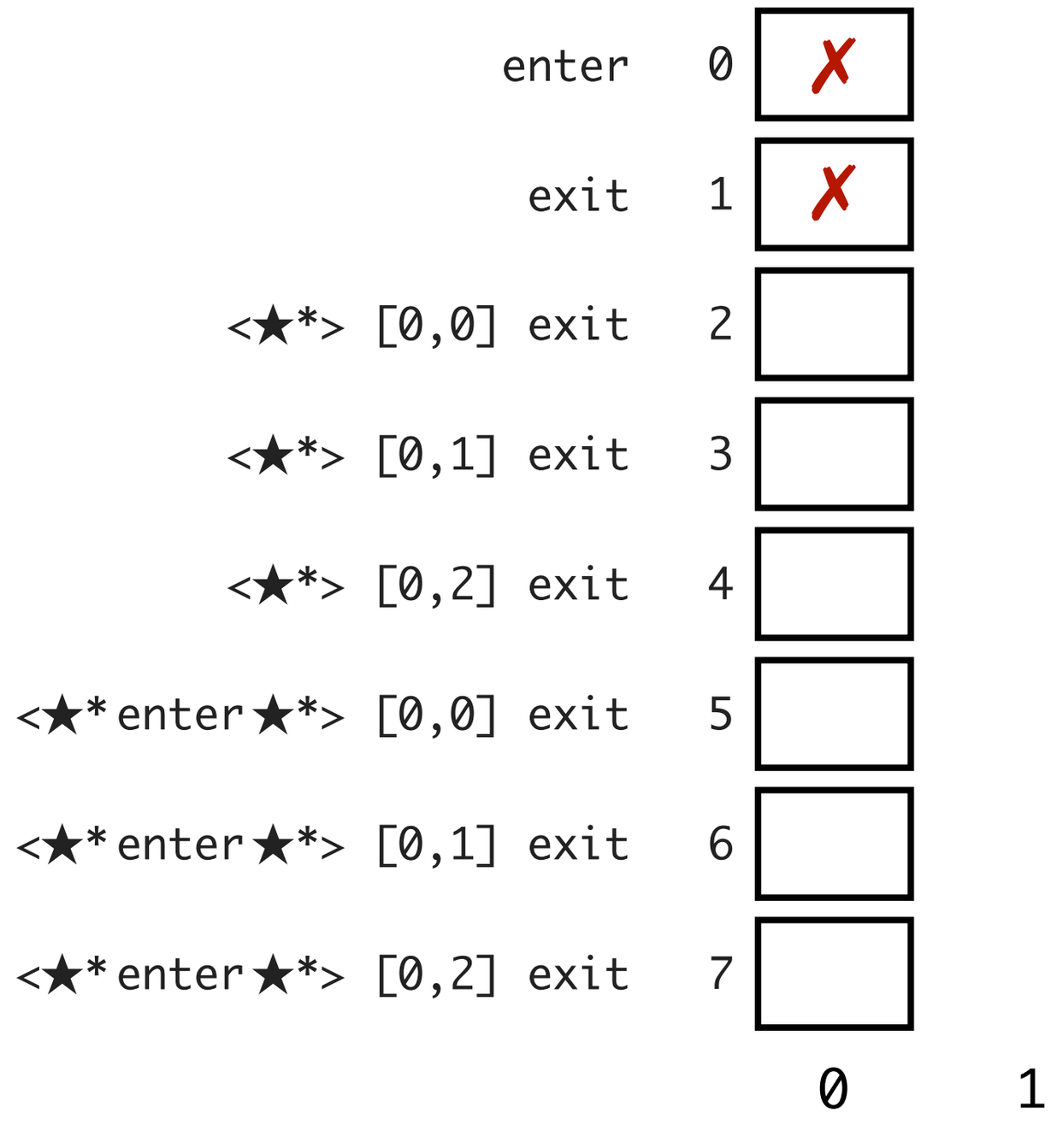
MDL Monitor by Example

```
                    enter   0
                     exit   1
          <★*> [0,0] exit   2
          <★*> [0,1] exit   3
          <★*> [0,2] exit   4
 <★*enter★*> [0,0] exit   5
 <★*enter★*> [0,1] exit   6
 <★*enter★*> [0,2] exit   7
```

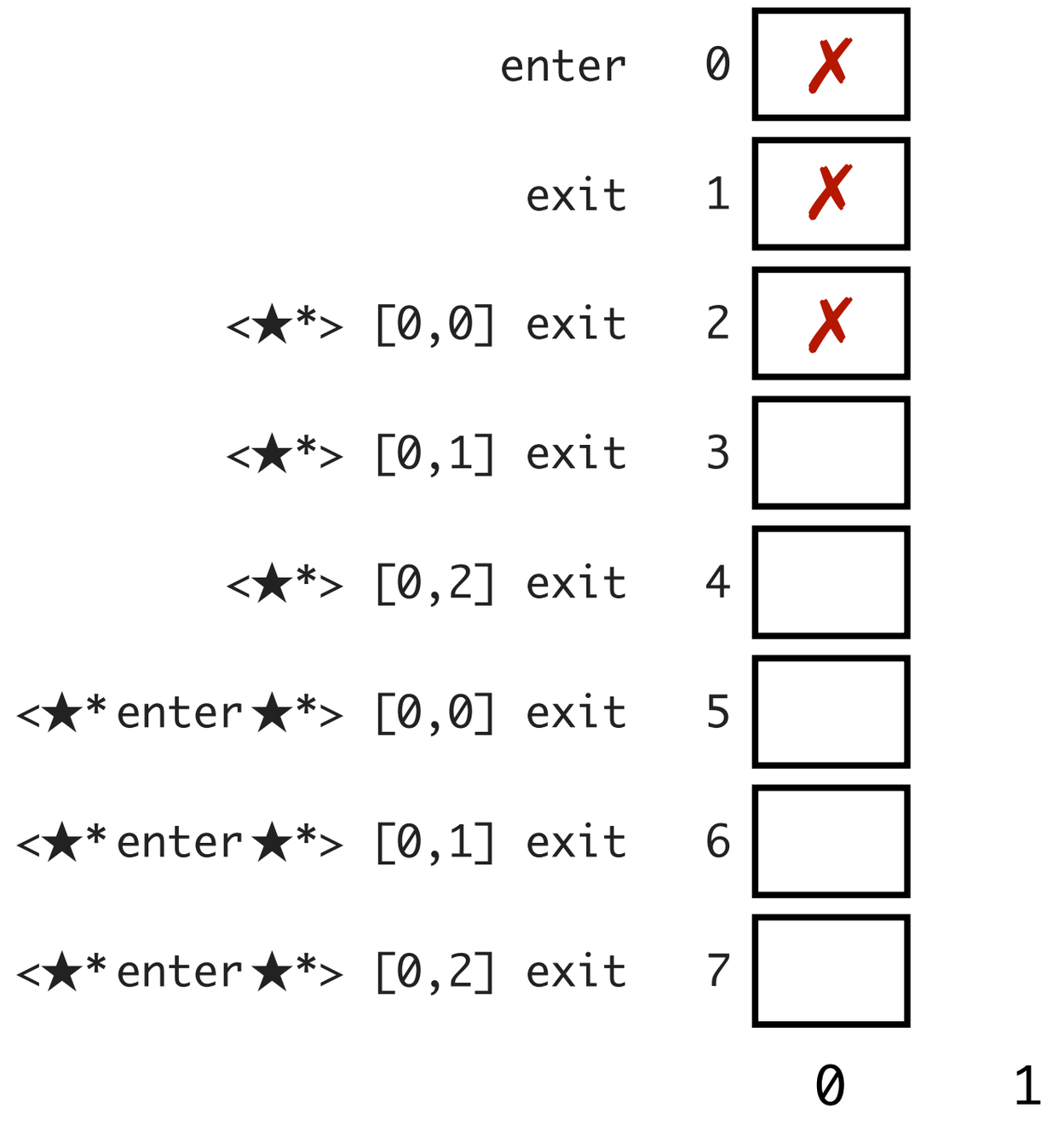
MDL Monitor by Example



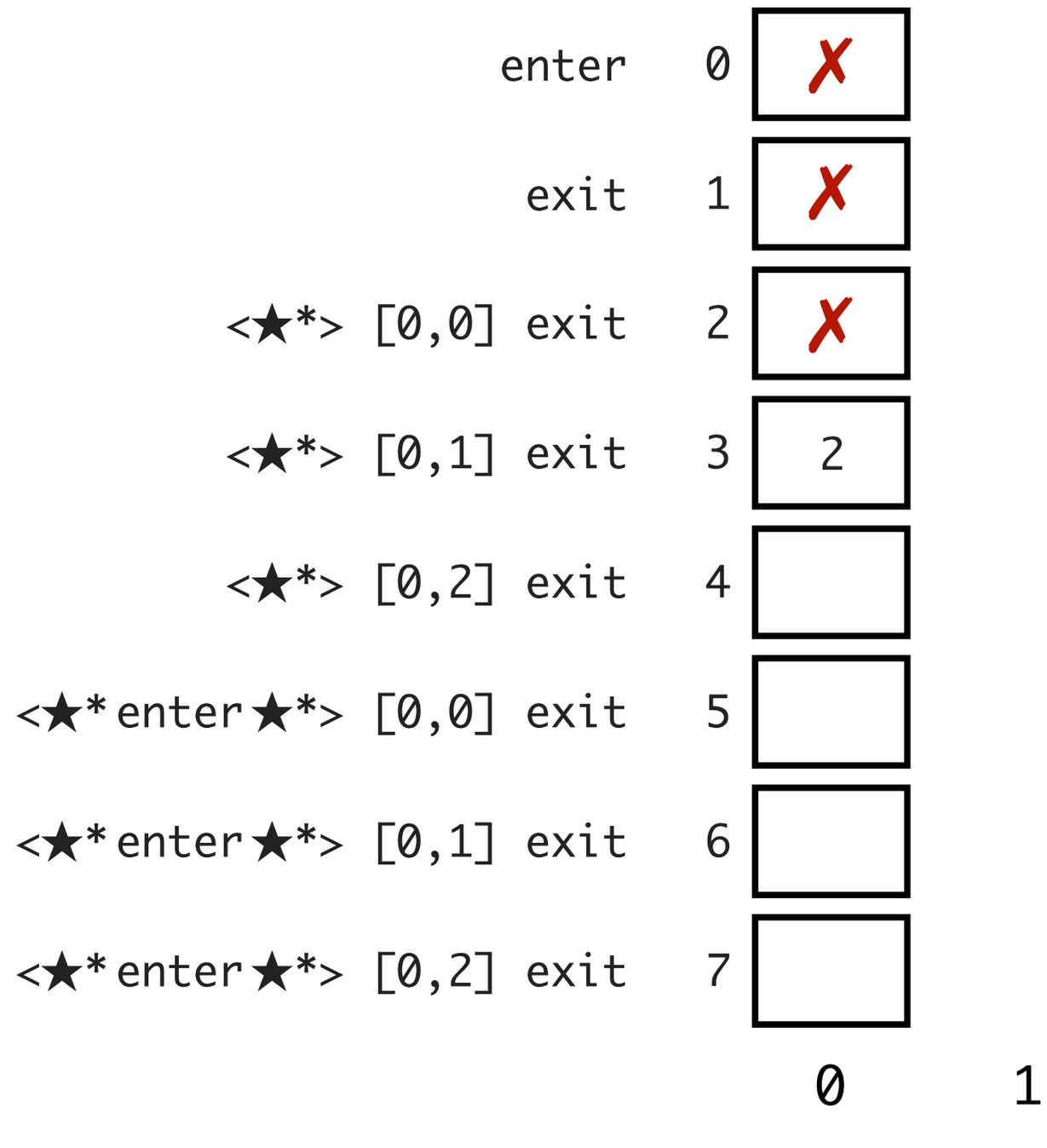
MDL Monitor by Example



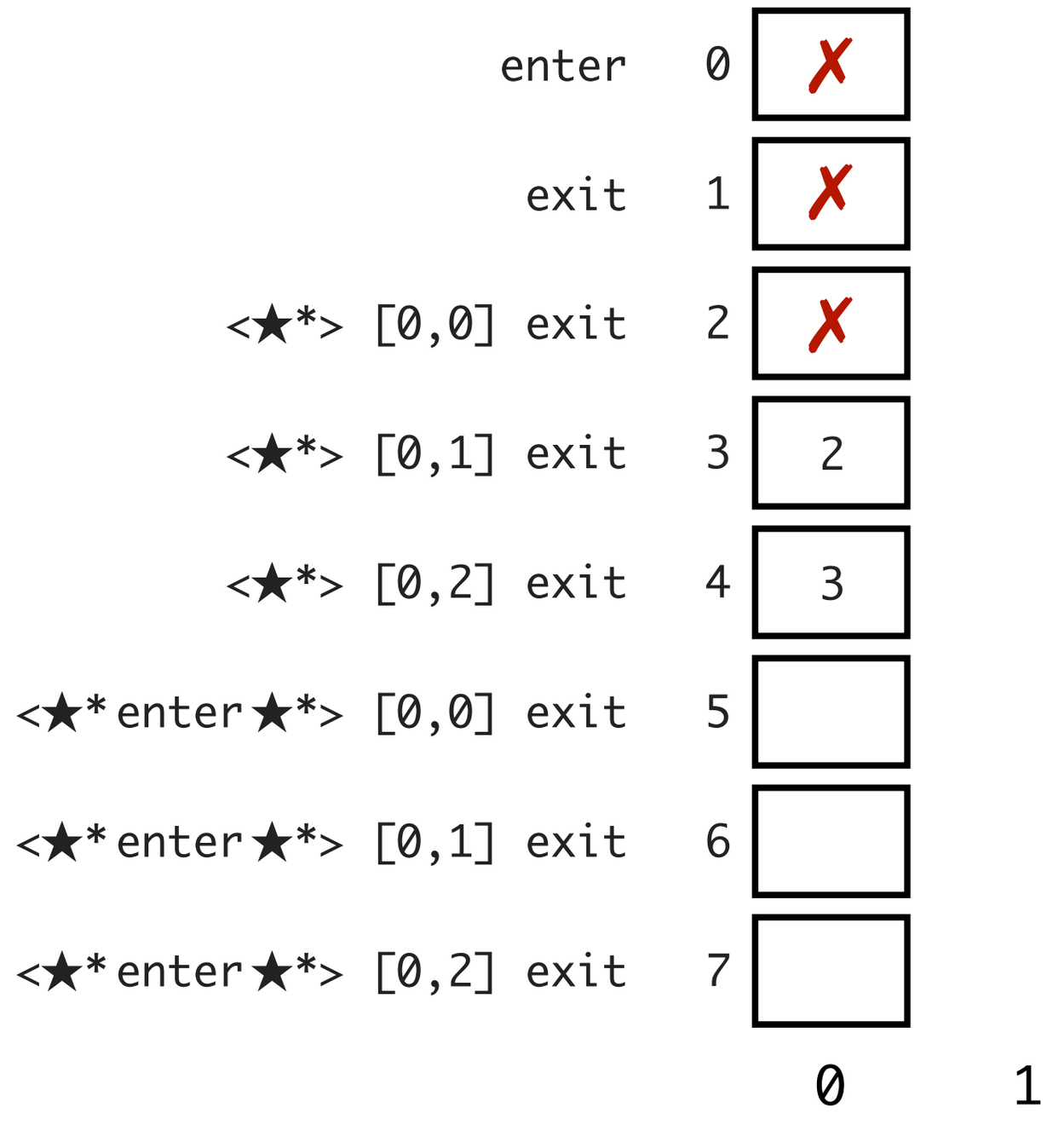
MDL Monitor by Example



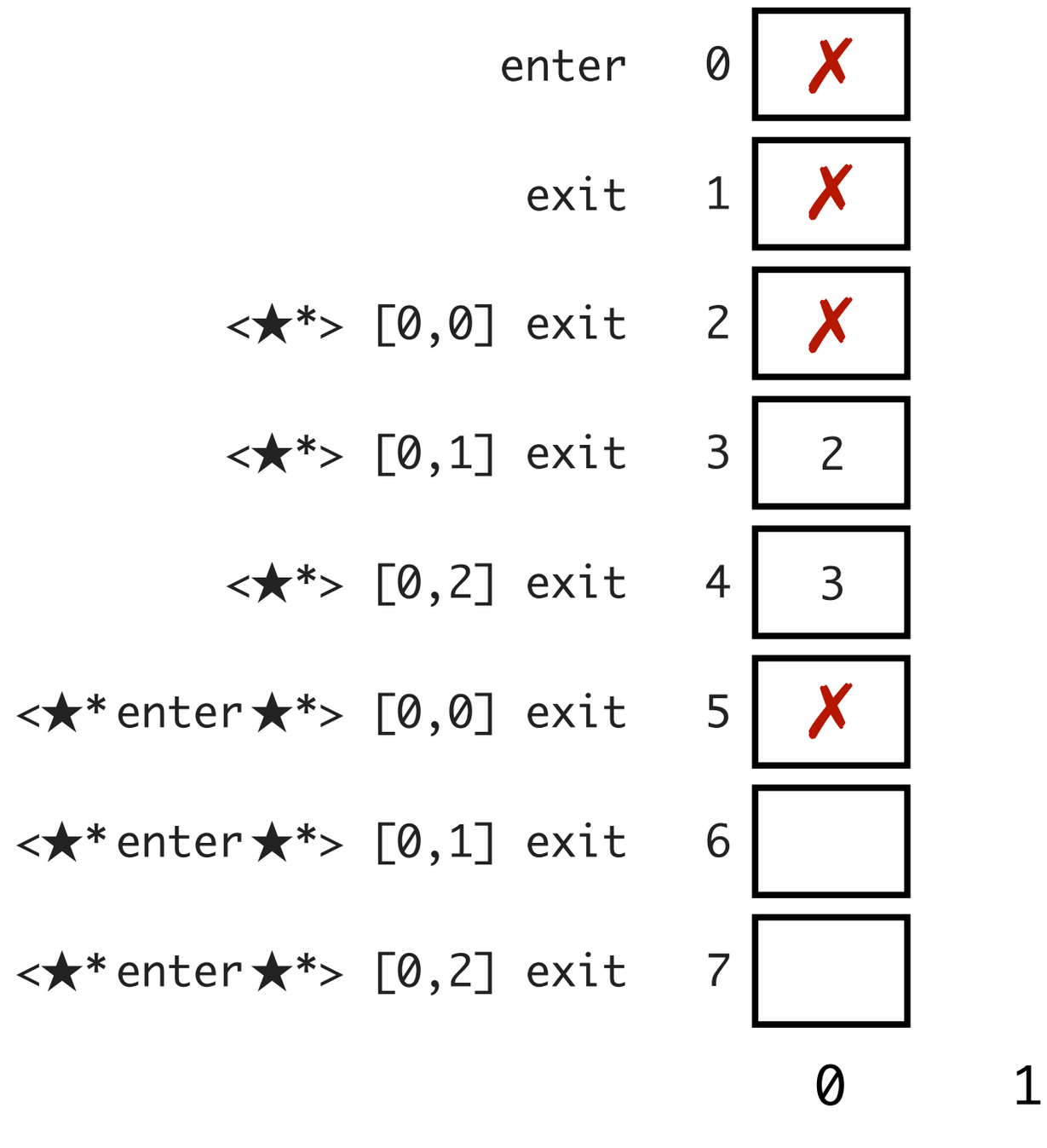
MDL Monitor by Example



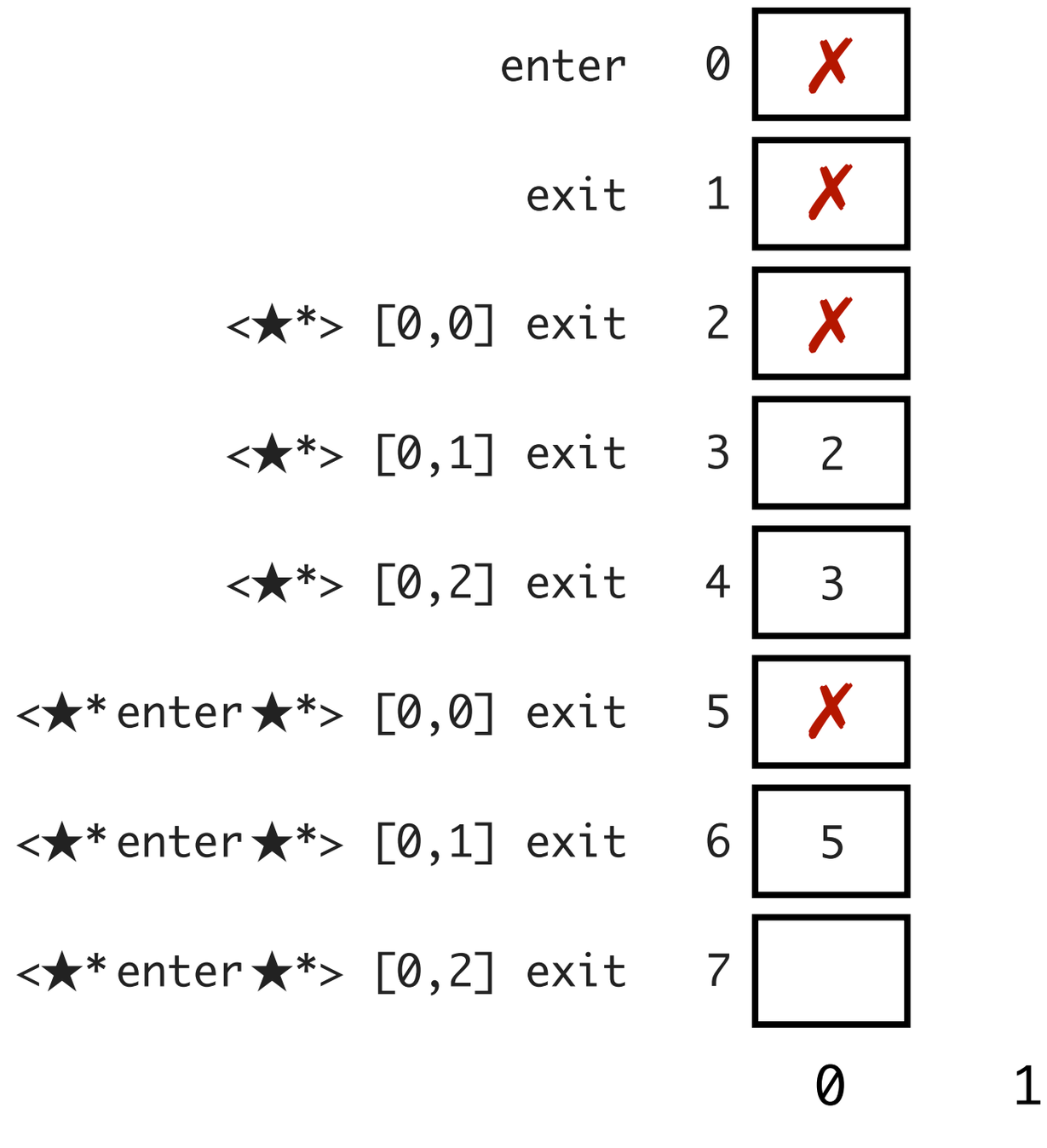
MDL Monitor by Example



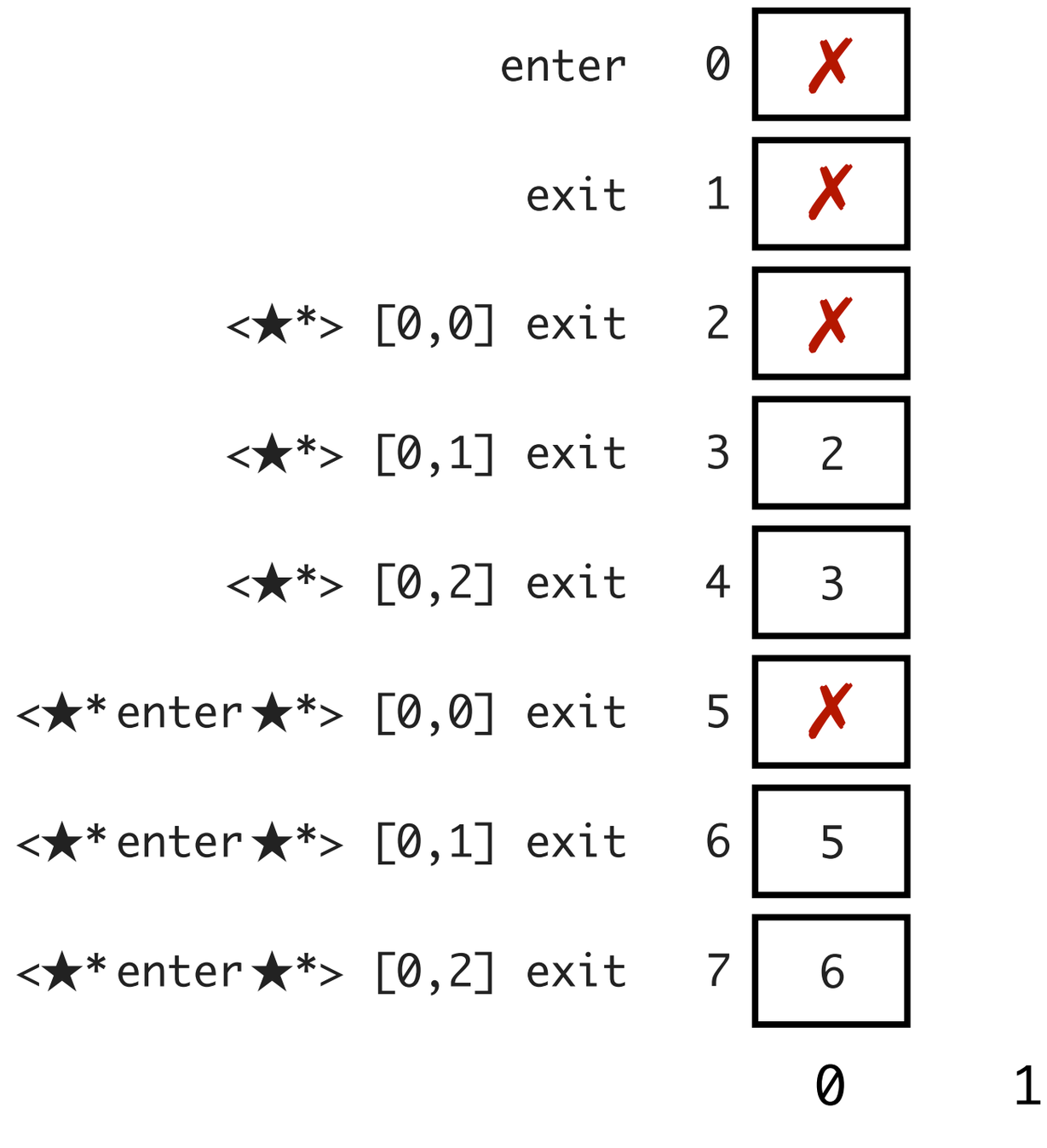
MDL Monitor by Example



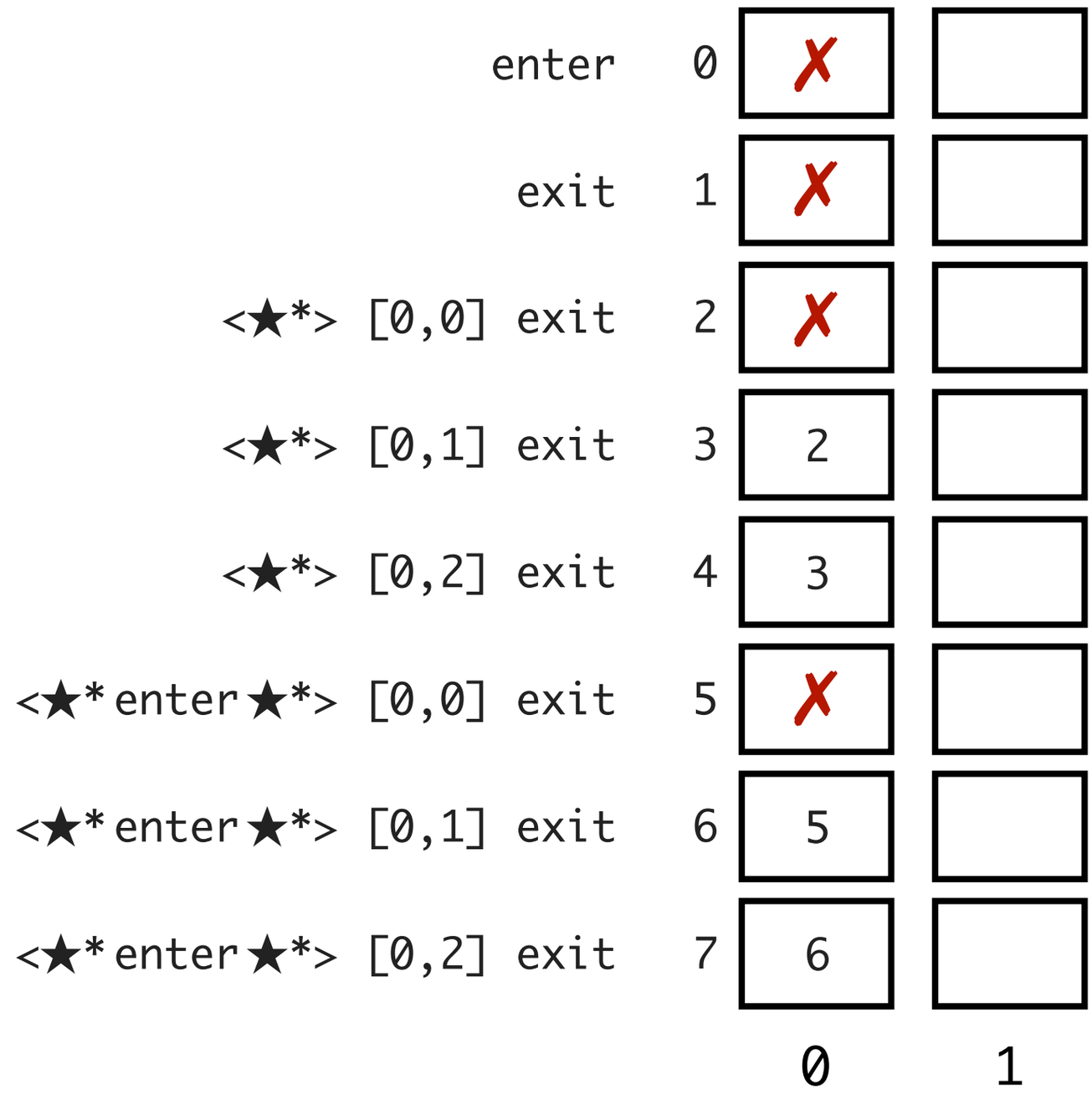
MDL Monitor by Example



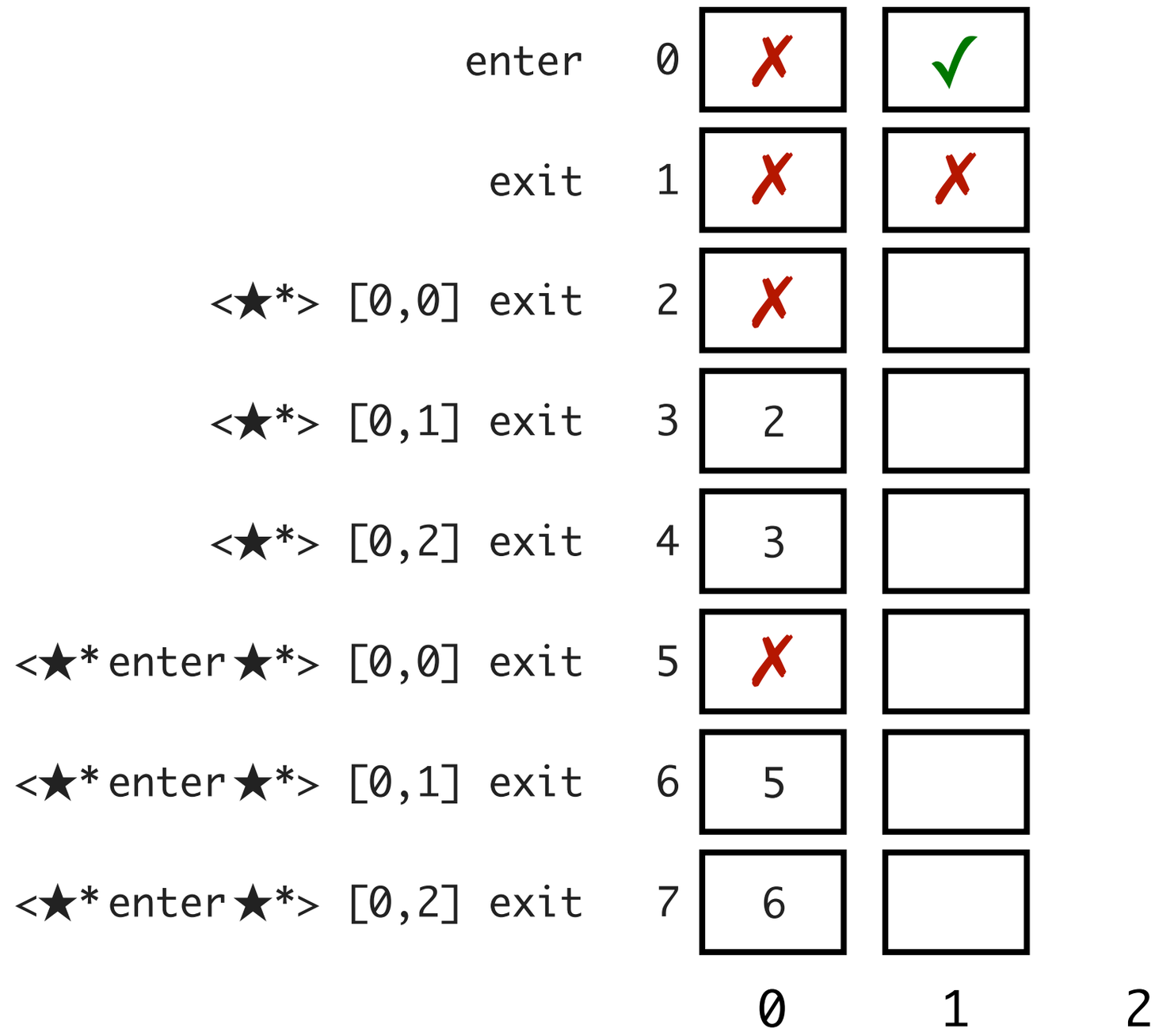
MDL Monitor by Example



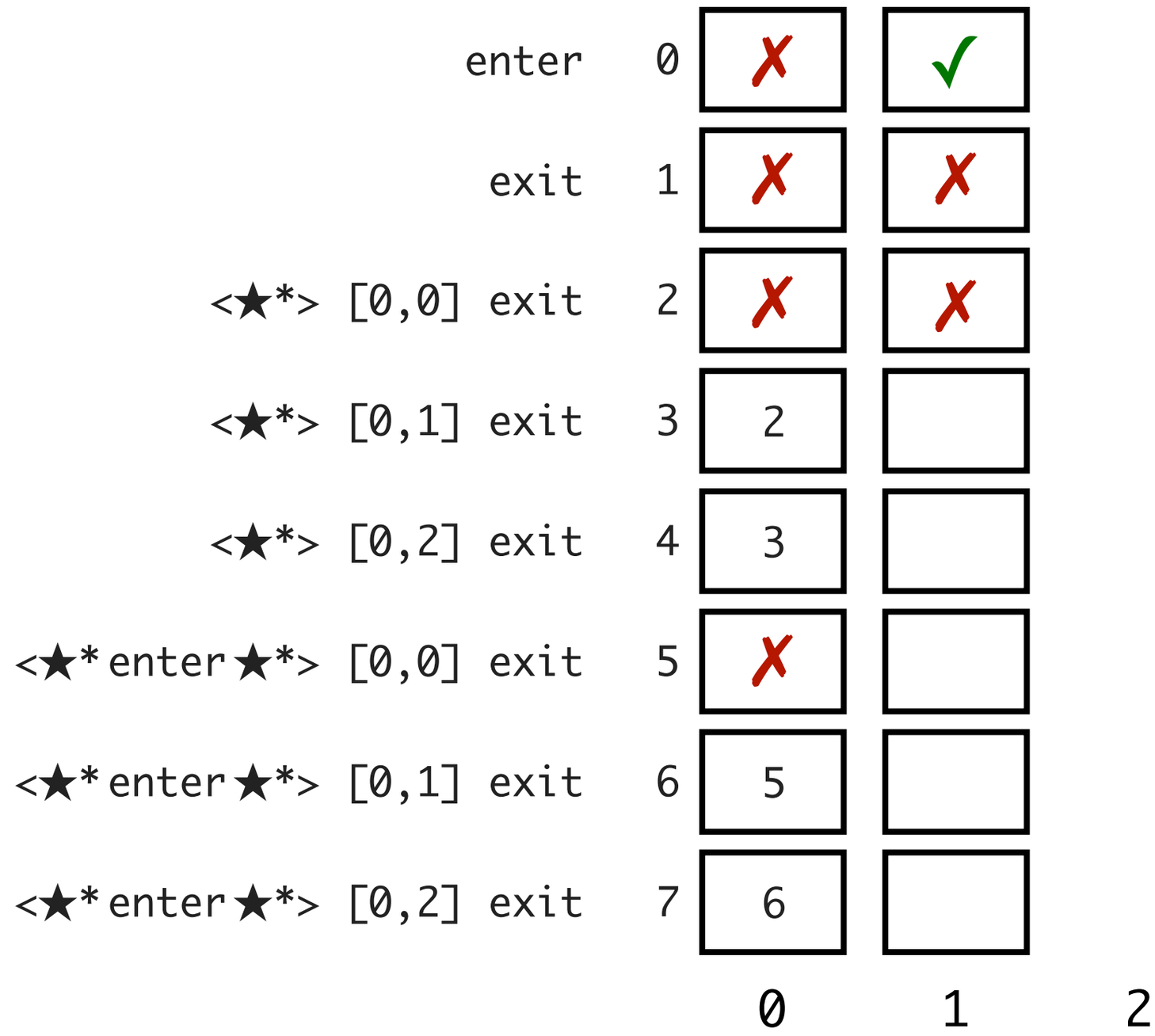
MDL Monitor by Example



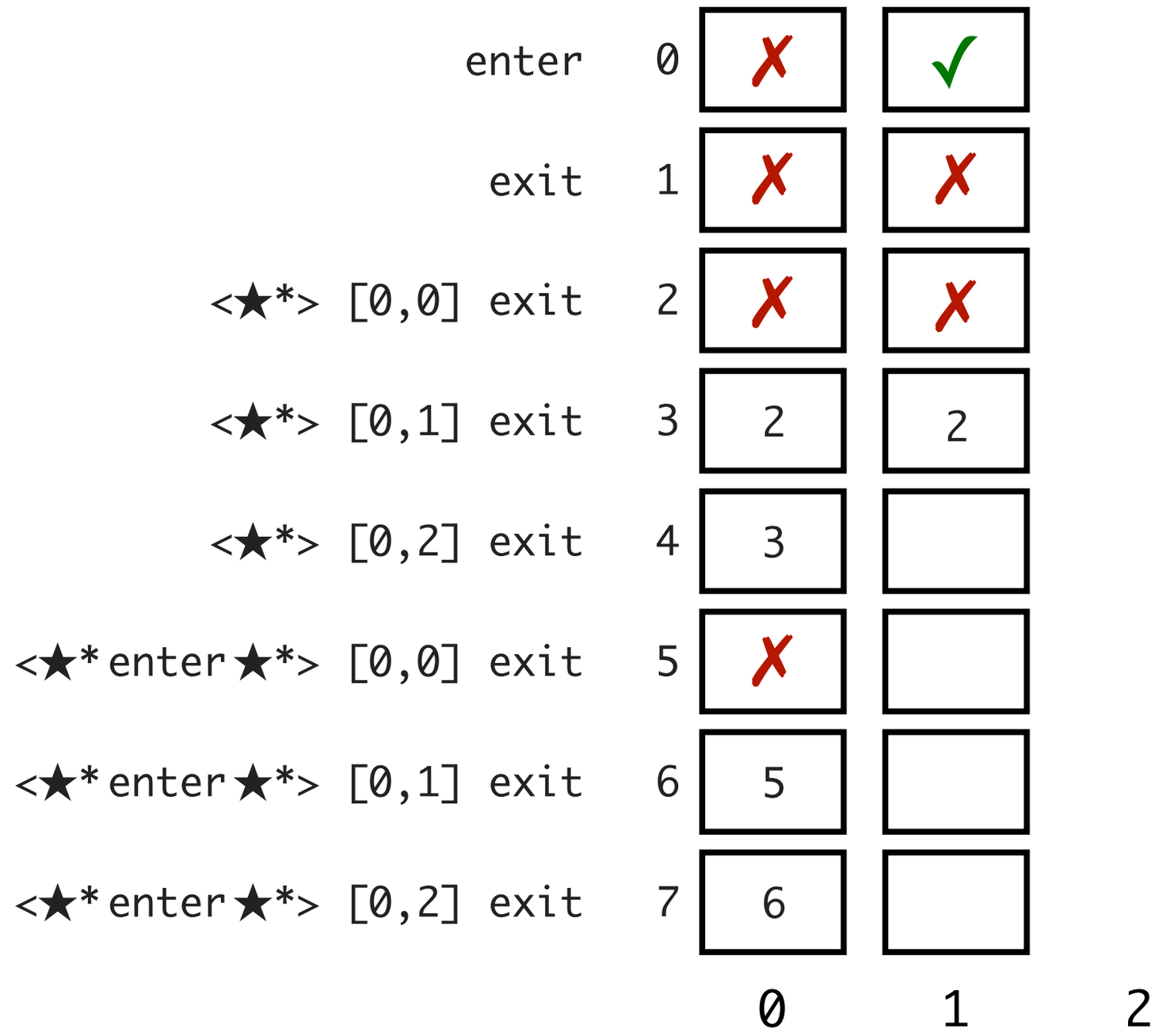
MDL Monitor by Example



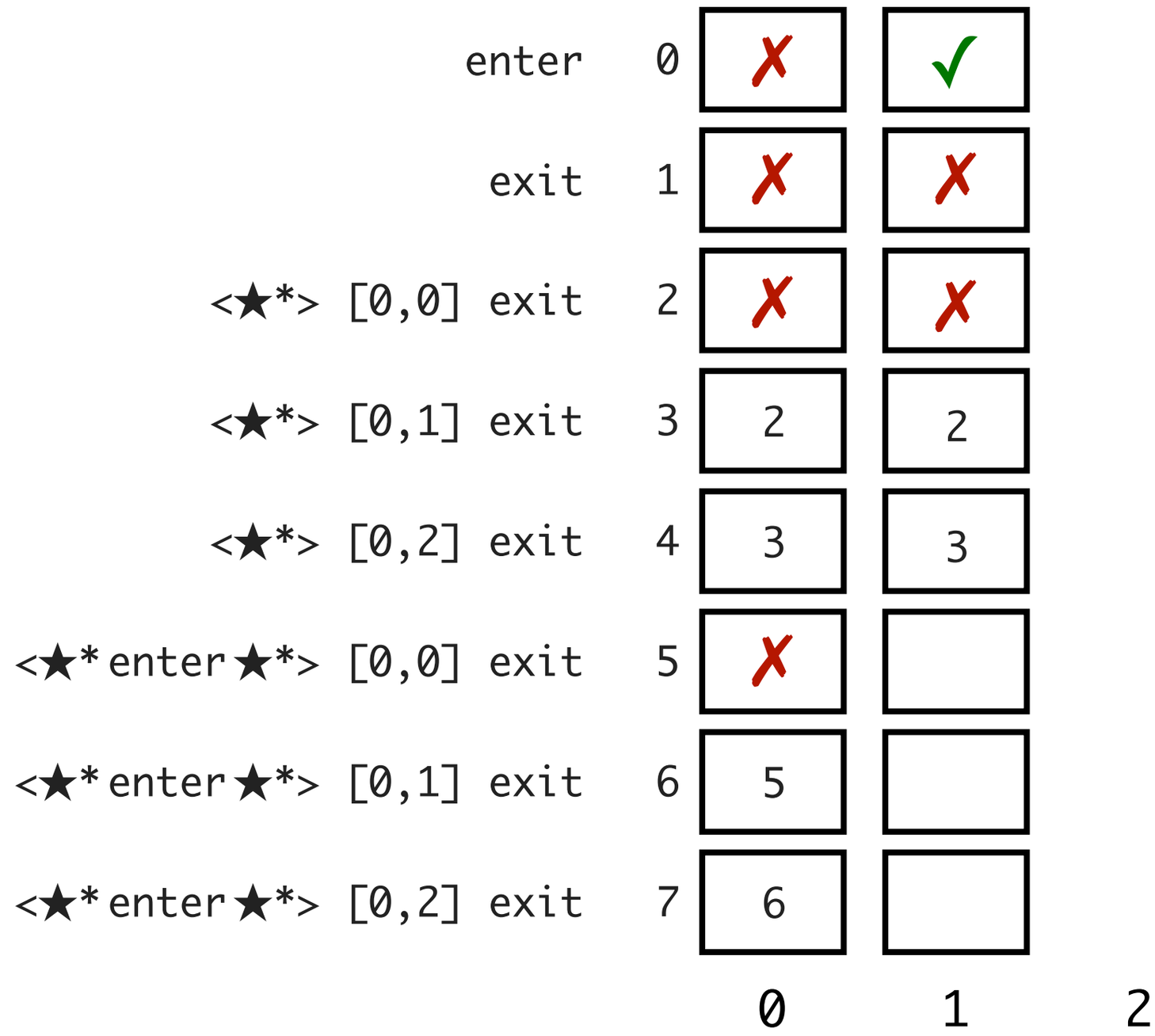
MDL Monitor by Example



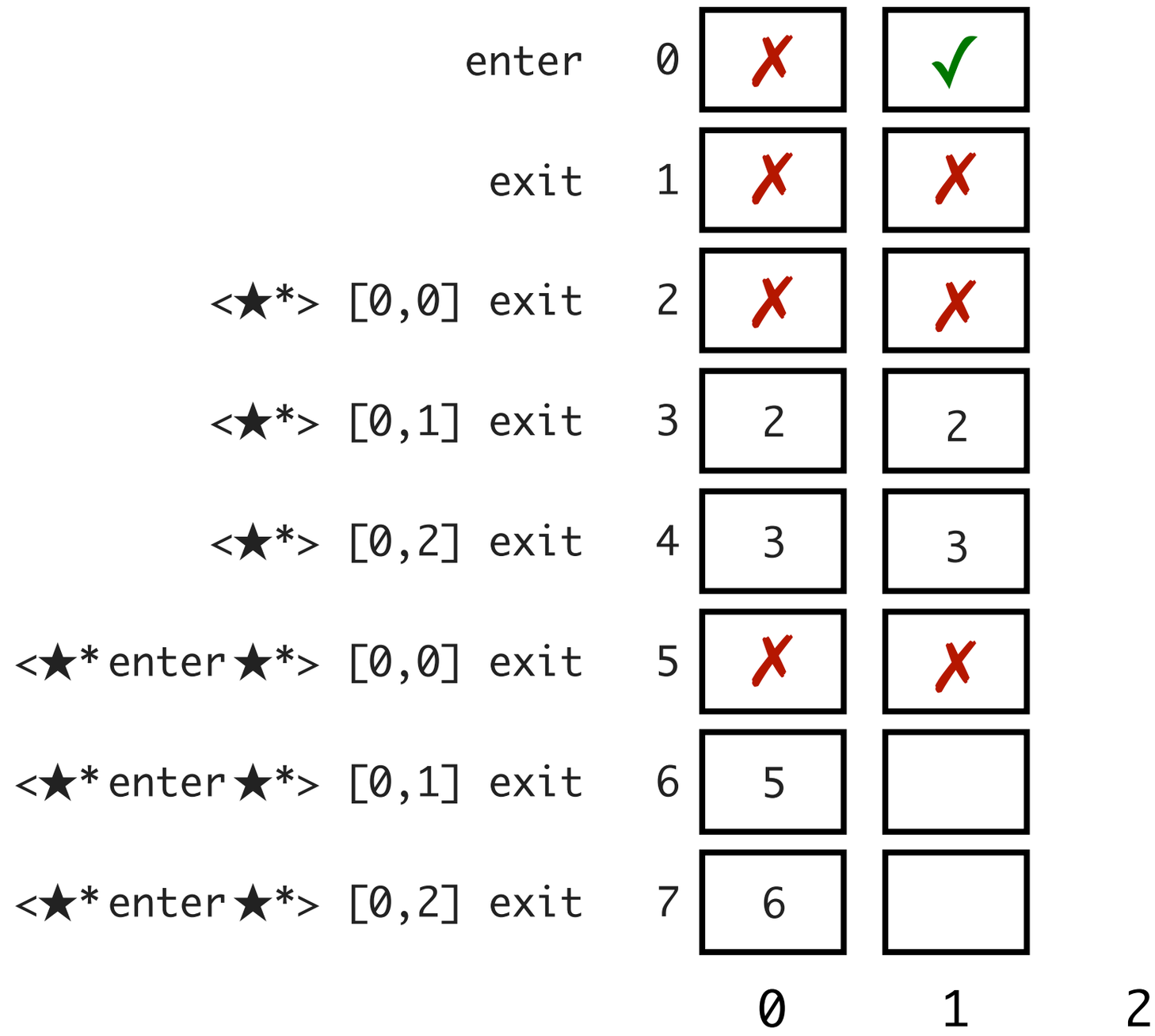
MDL Monitor by Example



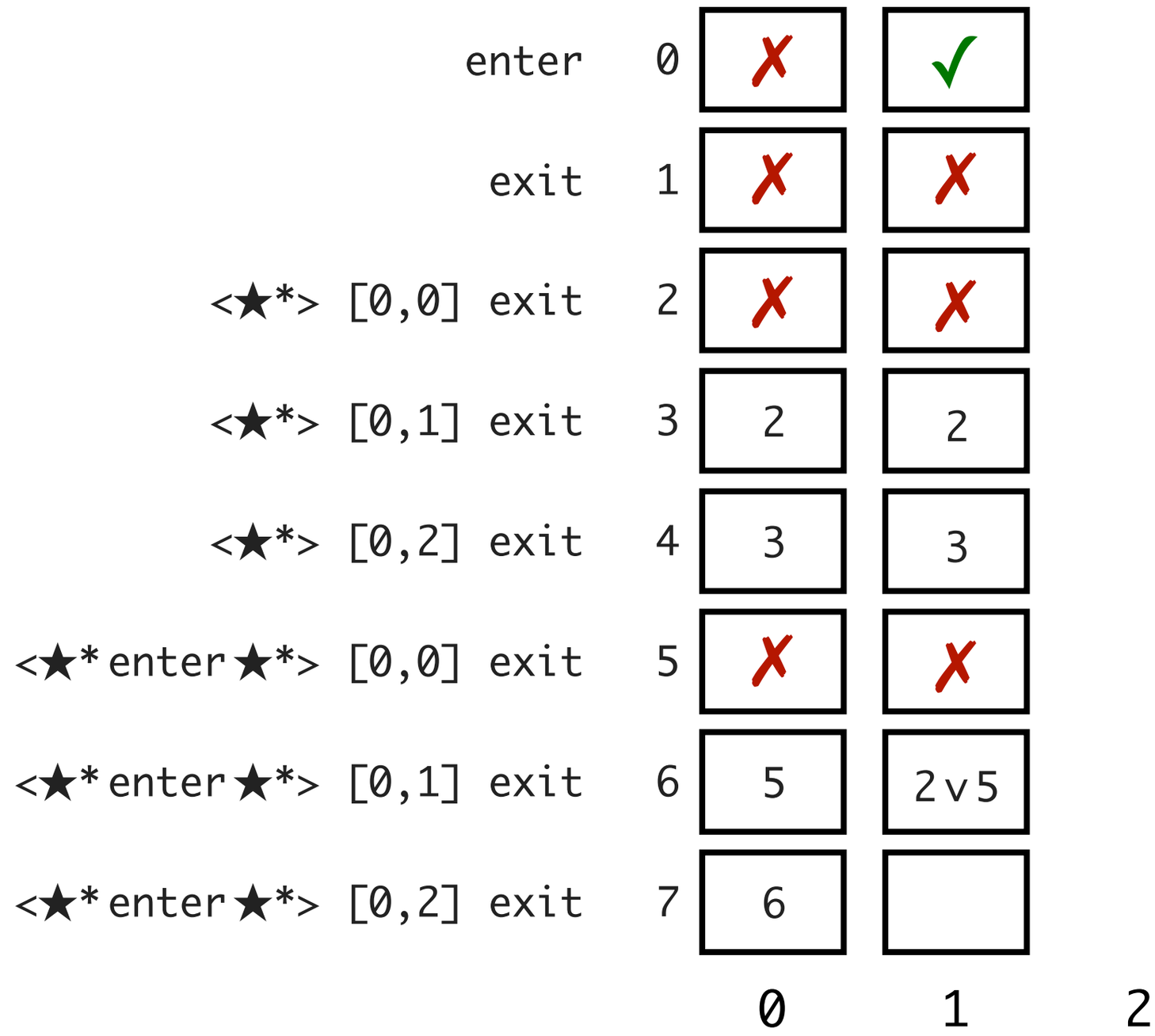
MDL Monitor by Example



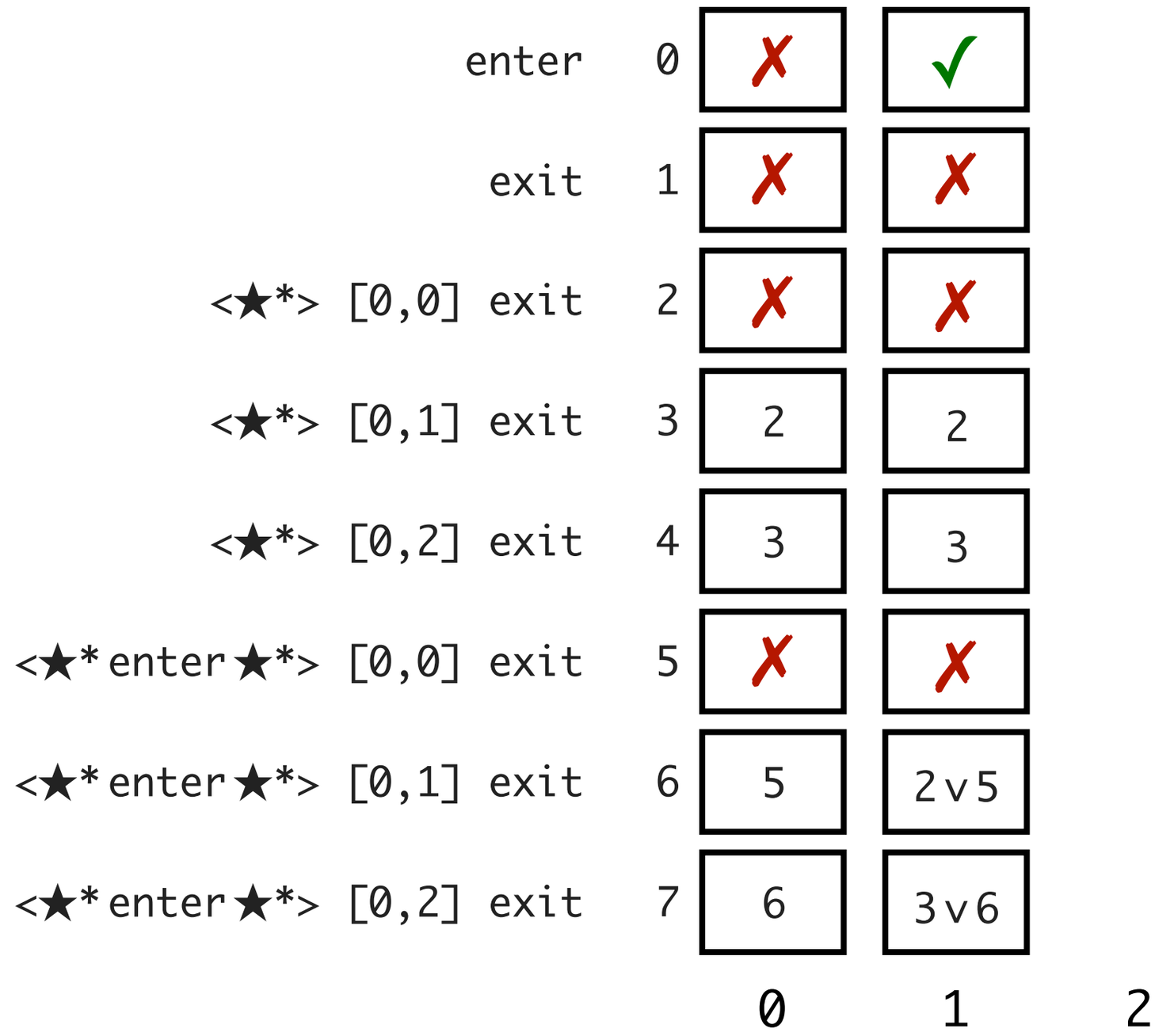
MDL Monitor by Example



MDL Monitor by Example



MDL Monitor by Example



MDL Monitor by Example

Formula		0	1	2
enter	0		✓	
exit	1		✗	
<★*> [0,0] exit	2		✗	
<★*> [0,1] exit	3		2	
<★*> [0,2] exit	4		3	
<★*enter★*> [0,0] exit	5		✗	
<★*enter★*> [0,1] exit	6		2v5	
<★*enter★*> [0,2] exit	7	6	3v6	

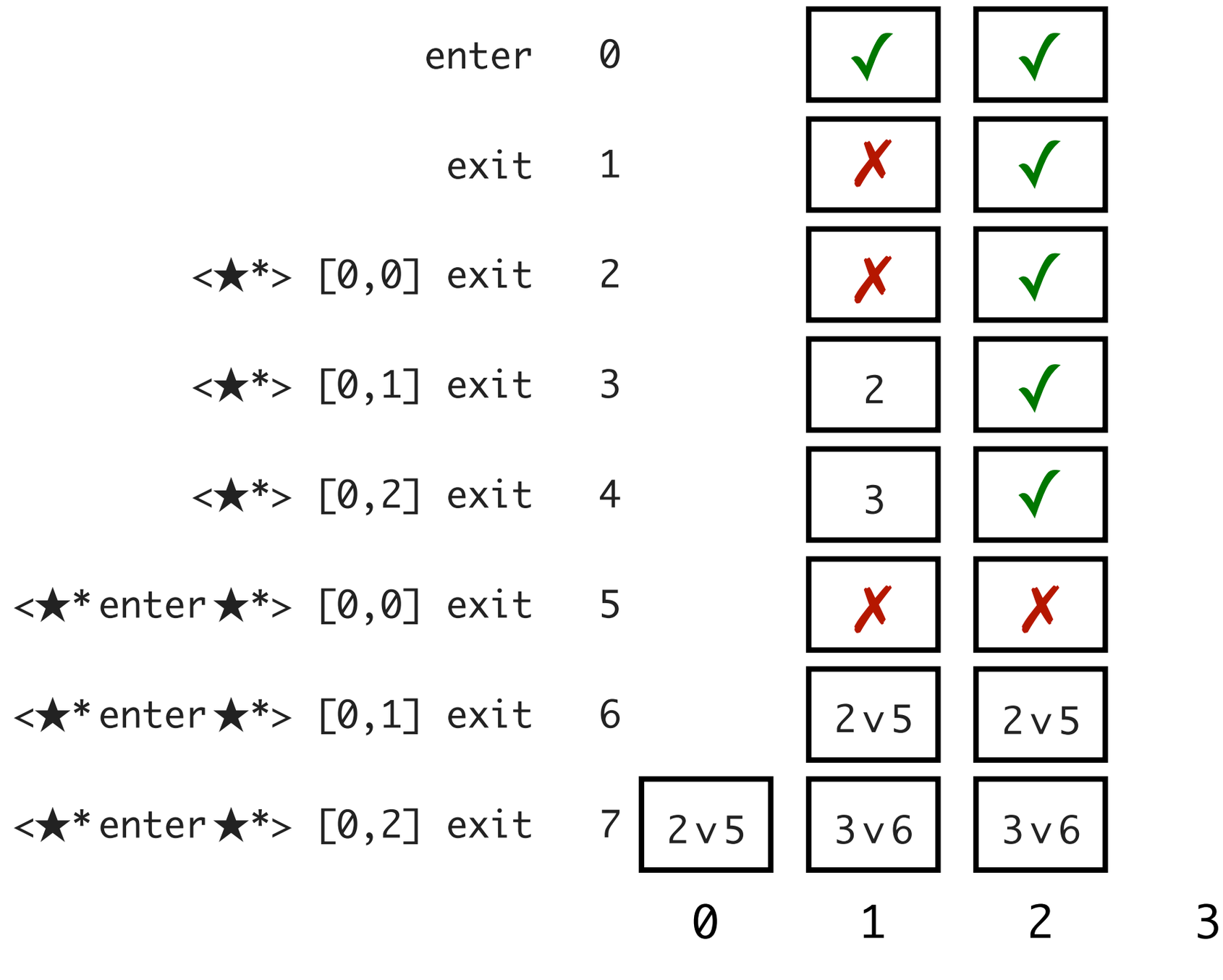
MDL Monitor by Example

		0	1	2
enter	0		✓	
exit	1		✗	
<★*> [0,0] exit	2		✗	
<★*> [0,1] exit	3		2	
<★*> [0,2] exit	4		3	
<★*enter★*> [0,0] exit	5		✗	
<★*enter★*> [0,1] exit	6		2v5	
<★*enter★*> [0,2] exit	7	2v5	3v6	

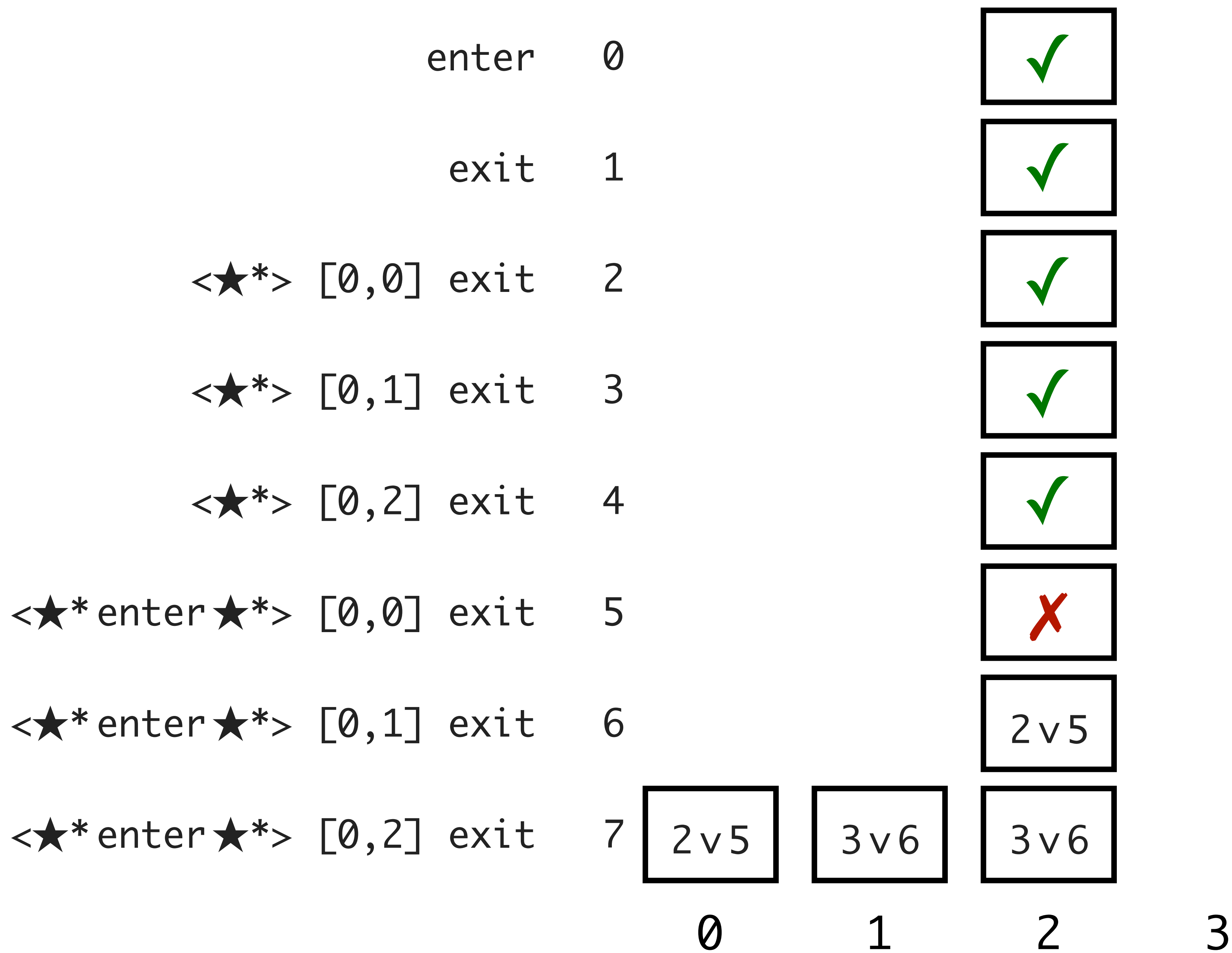
MDL Monitor by Example

		0	1	2	3
enter	0		✓		
exit	1		✗		
<★*> [0,0] exit	2		✗		
<★*> [0,1] exit	3		2		
<★*> [0,2] exit	4		3		
<★* enter ★*> [0,0] exit	5		✗		
<★* enter ★*> [0,1] exit	6		2v5		
<★* enter ★*> [0,2] exit	7	2v5	3v6		

MDL Monitor by Example

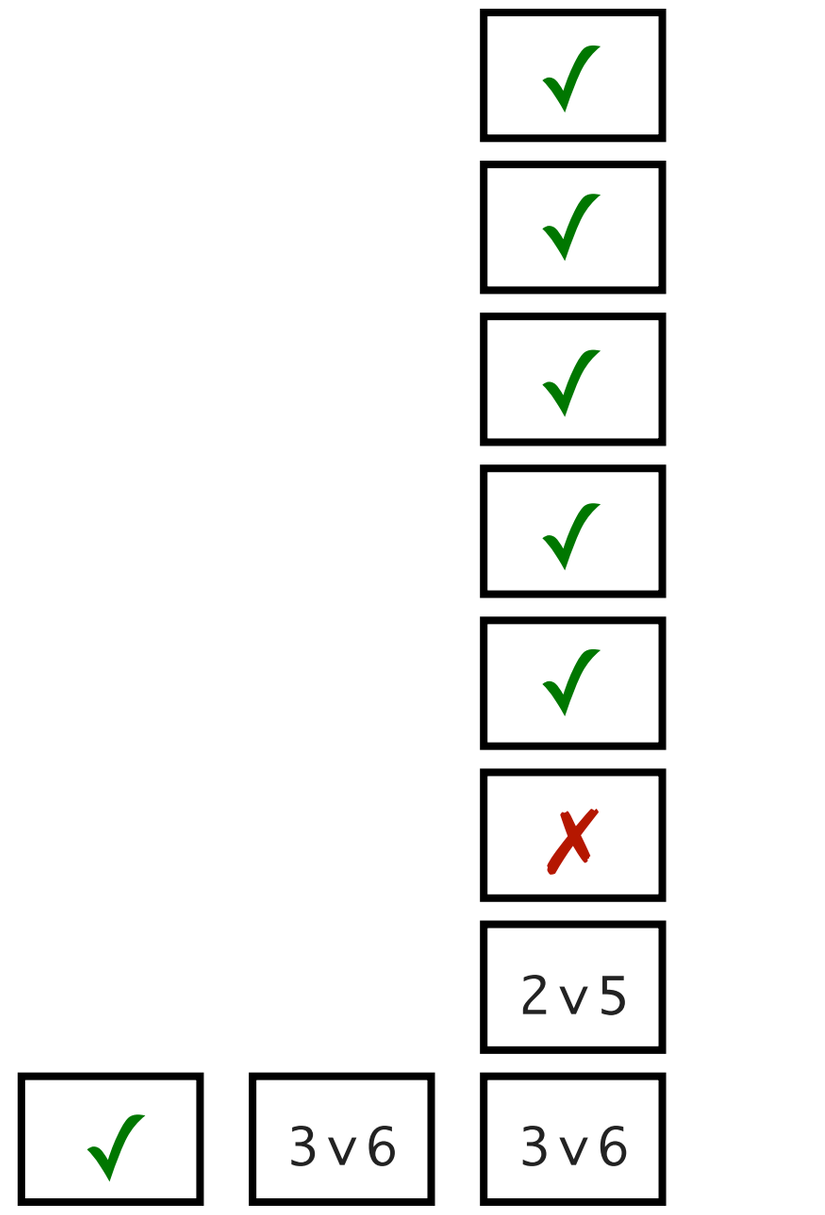


MDL Monitor by Example



MDL Monitor by Example

`enter`	`0`		
`exit`	`1`		
`<★*> [0,0] exit`	`2`		
`<★*> [0,1] exit`	`3`		
`<★*> [0,2] exit`	`4`		
`<★*enter★*> [0,0] exit`	`5`		
`<★*enter★*> [0,1] exit`	`6`		
`<★*enter★*> [0,2] exit`	`7`		

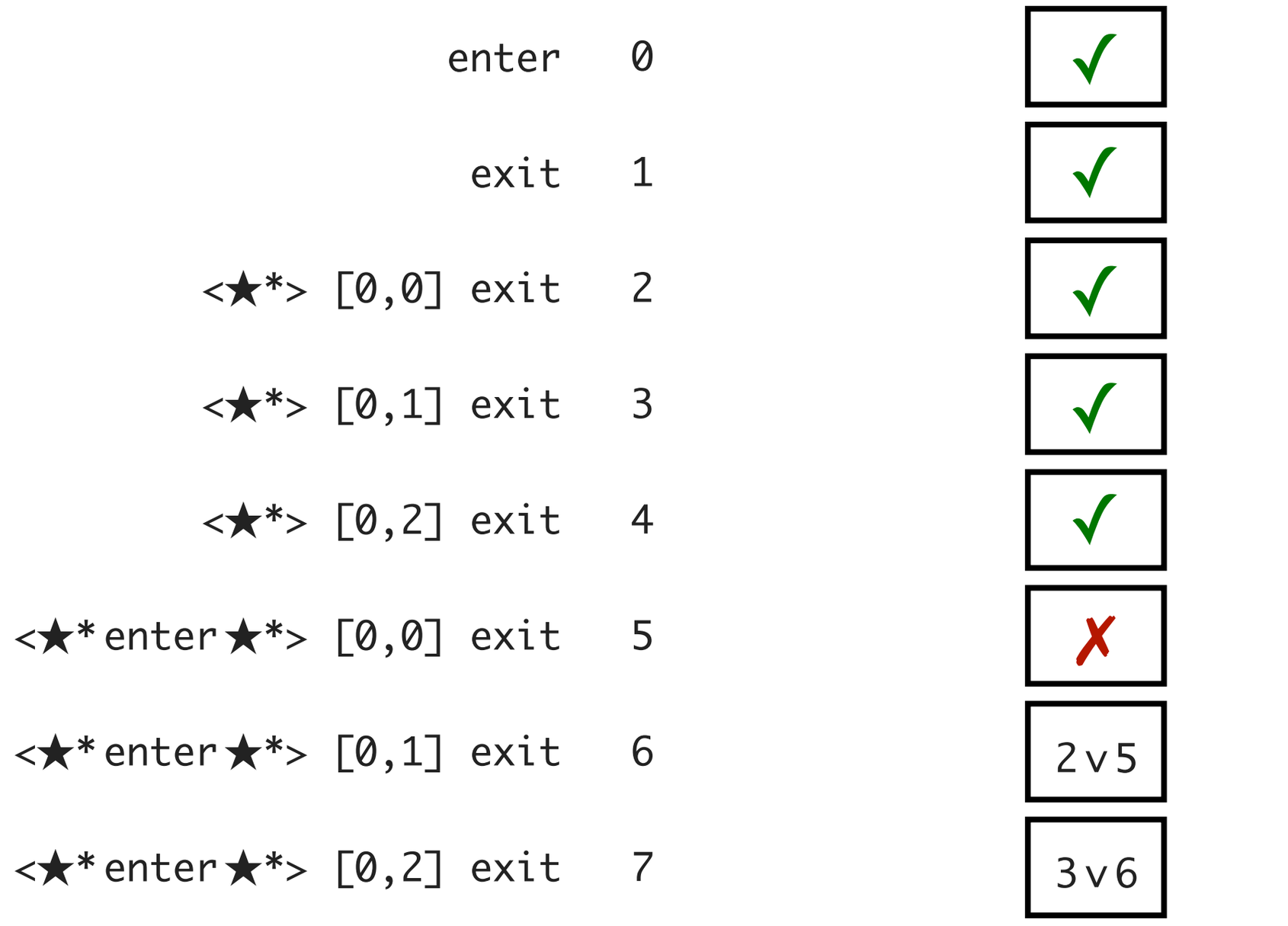


MDL Monitor by Example

`enter`	`0`	✓	
`exit`	`1`	✓	
`<★*>` `[0,0]` `exit`	`2`	✓	
`<★*>` `[0,1]` `exit`	`3`		✓
`<★*>` `[0,2]` `exit`	`4`		✓
`<★*enter★*>` `[0,0]` `exit`	`5`		✗
`<★*enter★*>` `[0,1]` `exit`	`6`		2v5
`<★*enter★*>` `[0,2]` `exit`	`7`	✓	✓	3v6

`0` `1` `2` `3`

MDL Monitor by Example



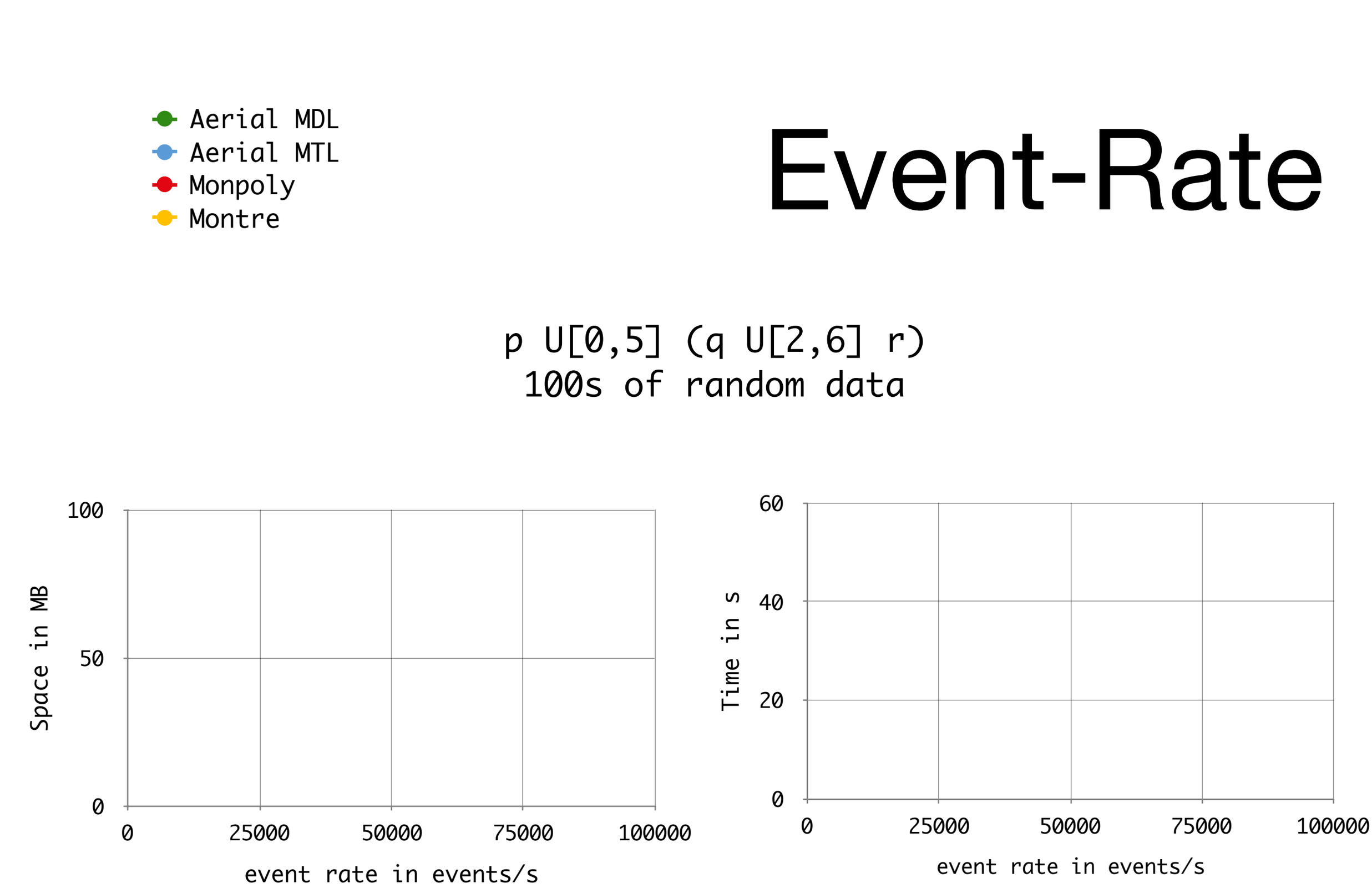
Evaluation

Event-Rate

```
p U[0,5] (q U[2,6] r)
 100s of random data
```

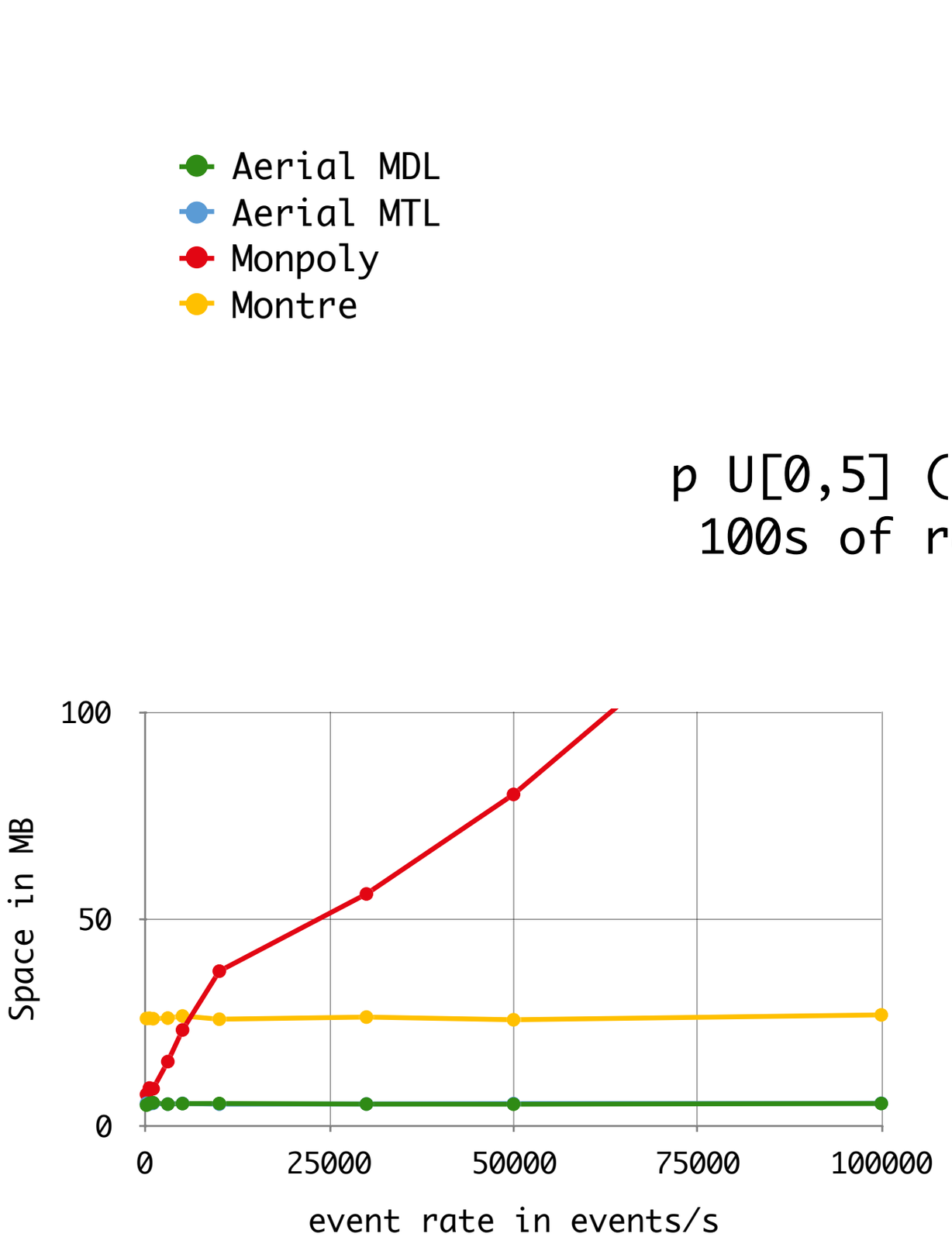
Event-Rate

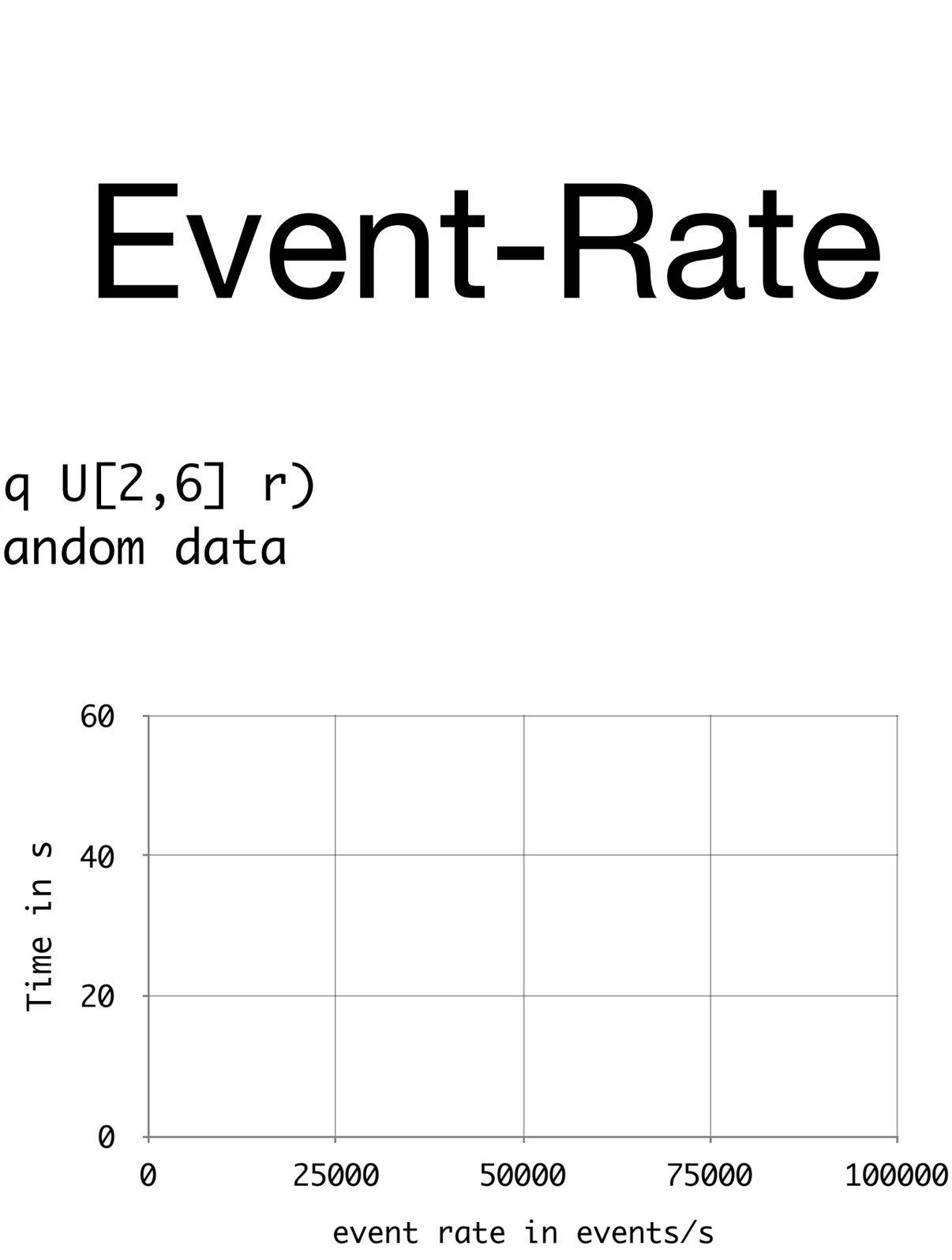
p U[0,5] (q U[2,6] r)
100s of random data



Event-Rate

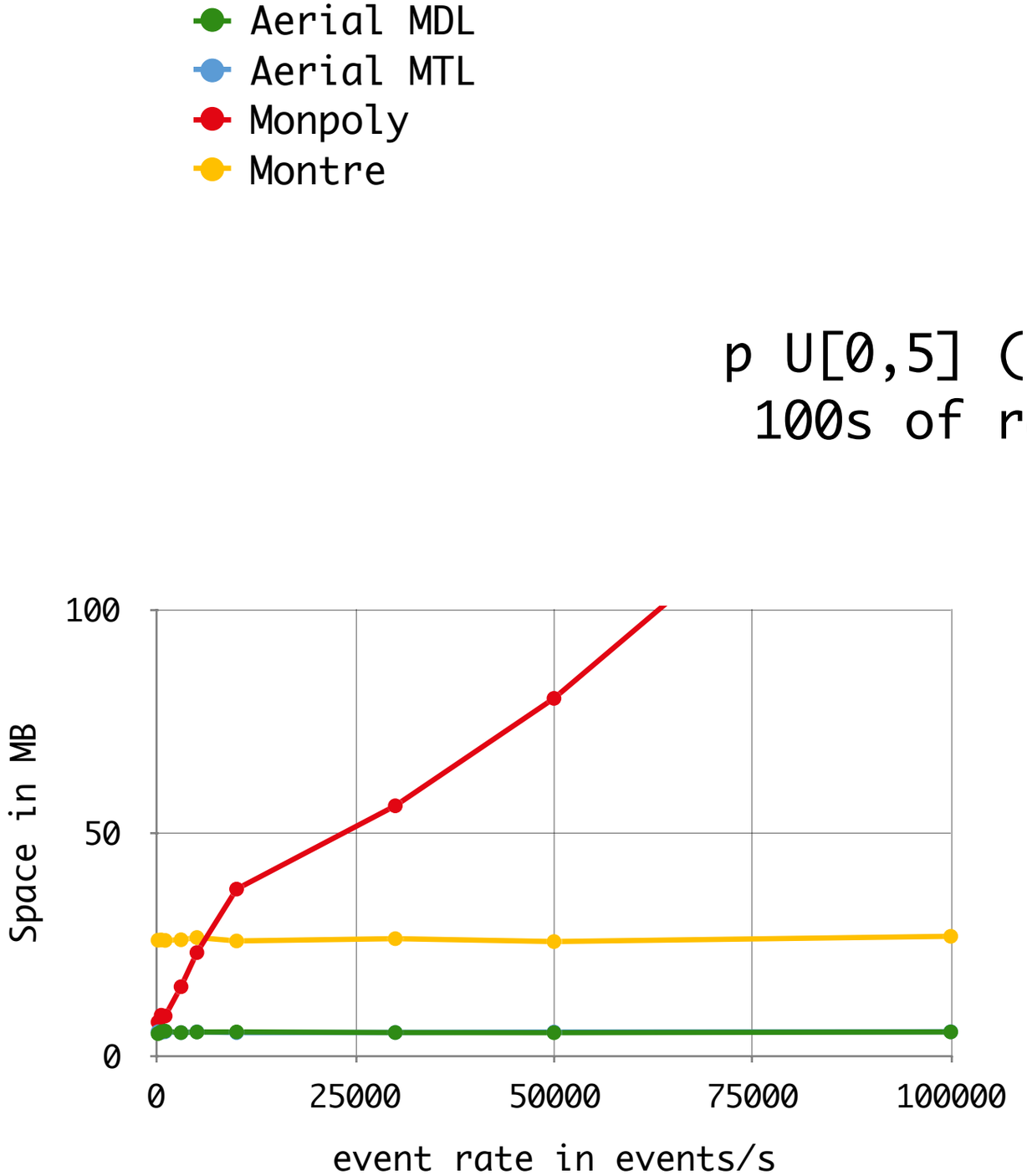
p U[0,5] (q U[2,6] r)
100s of random data

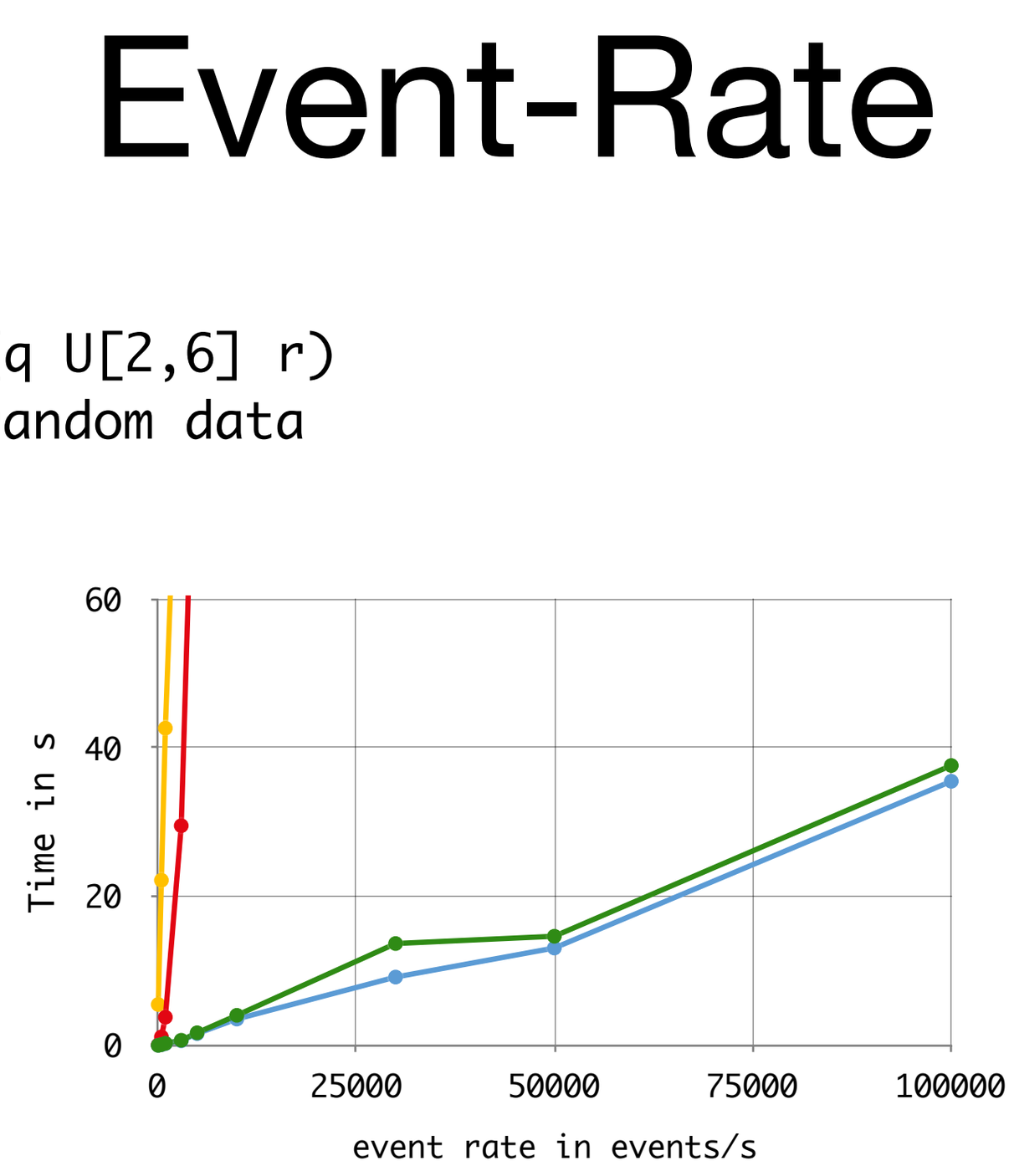




Event-Rate

p U[0,5] (q U[2,6] r)
100s of random data



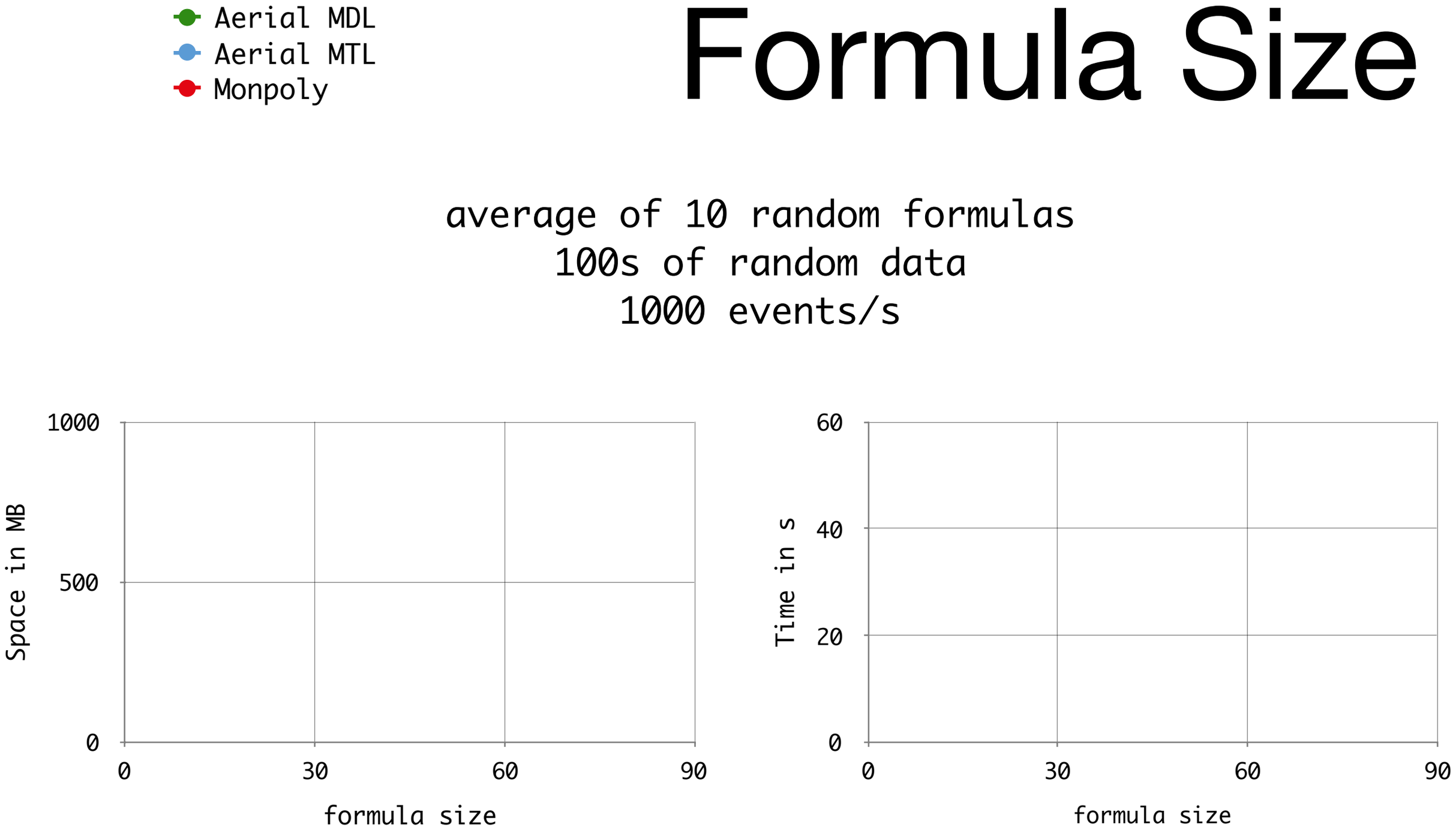


Formula Size

average of 10 random formulas
100s of random data
1000 events/s

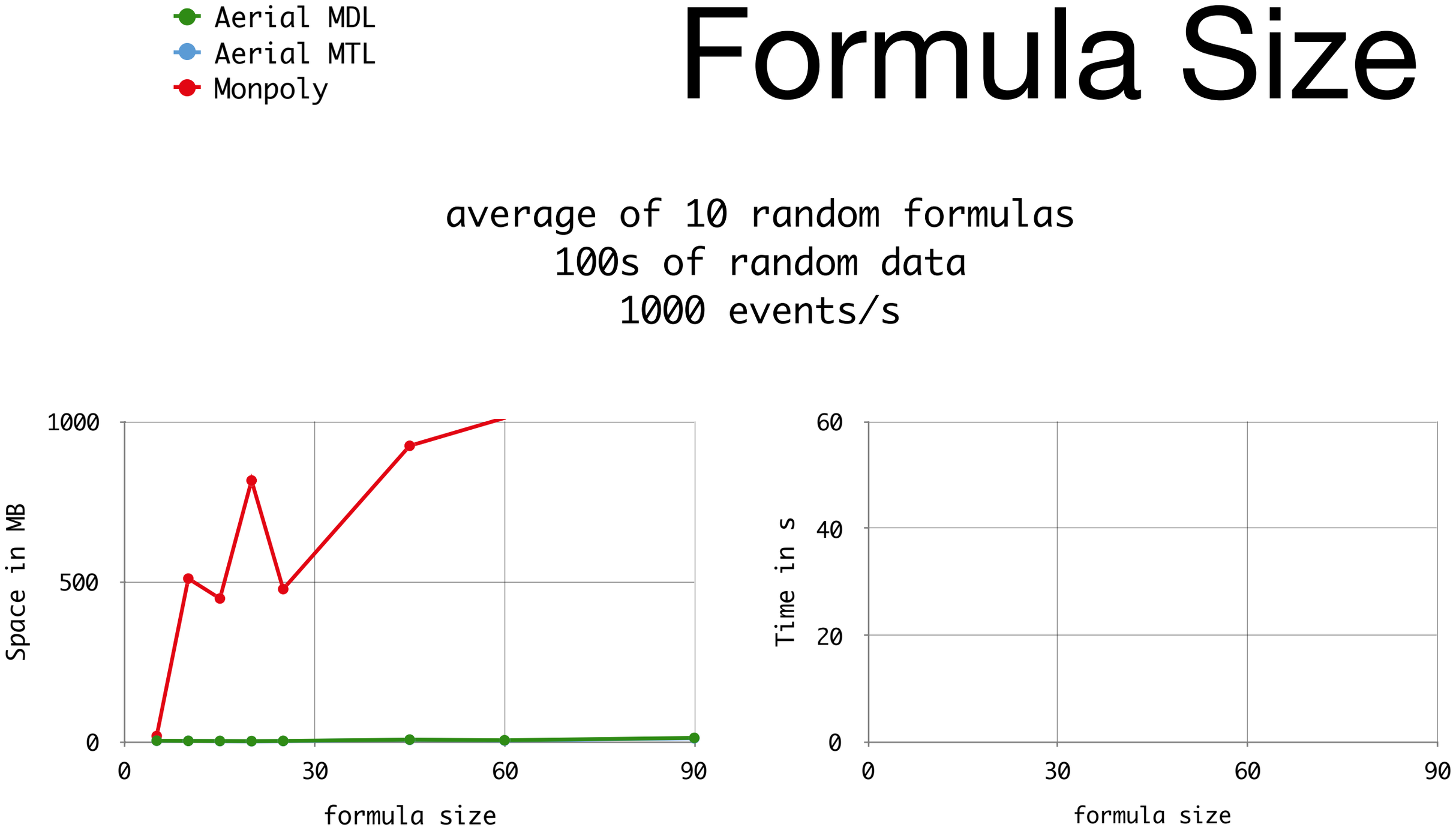
Formula Size

average of 10 random formulas
100s of random data
1000 events/s



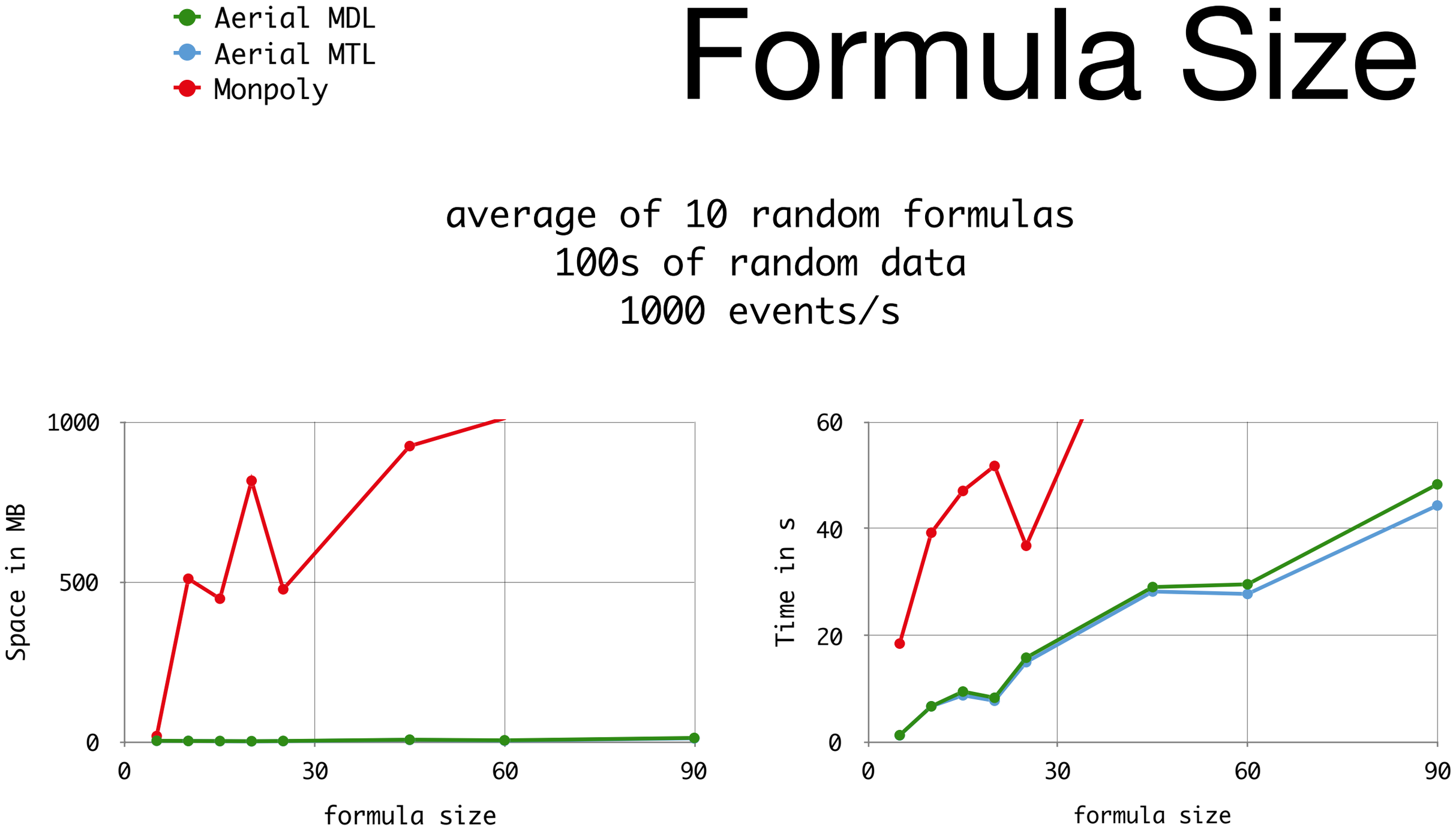
Formula Size

average of 10 random formulas
100s of random data
1000 events/s



Formula Size

average of 10 random formulas
100s of random data
1000 events/s



Future Work

Expressiveness

LTL
Pnueli
1977

MTL
Koymans
1990

LDL
De Giacomo, Vardi
2013

MDL
This paper

add regular expressions

add notion of time

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MFODL

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Almost Event-Rate Independent Monitoring of Metric Dynamic Logic

David Basin Srđan Krstić Dmitriy Traytel

Thanks!
Questions?

ETH zürich

75 NRP **Big Data**
National Research Programme